Increasing/Decreasing Functions

Consider a function $f(x)$, defined between numbers a and b. (So $a < x < b$.)

We say $f(x)$ is increasing if when $x_1 < x_2$ we have $f(x_1) < f(x_2)$ and

$f(x)$ is decreasing if when $x_1 < x_2$ we have $f(x_1) > f(x_2)$.

Finding intervals where $f(x)$ is increasing and decreasing:

(1) Find the values of x where $f'(x) = 0$, or where $f'(x)$ is not continuous (or not defined). Then we get a collection of intervals $a < x < b$.

(2) For each interval choose a c in this interval. Evaluate $f'(c)$.

If $f'(c) > 0$ then the function $f(x)$ is increasing on this interval.

If $f'(c) < 0$ then the function $f(x)$ is decreasing on this interval.

NOTE: At values where $f(x)$ is not defined or continuous, then $f'(x)$ is not continuous.

Examples

Find the intervals where the following functions are increasing and decreasing.

$f_1(x) = 1 + 2x - x^2.$

$f_2(x) = x^3 - 2x^2 + 4x - 3.$

$f_2(x) = \frac{x^2-1}{2x-1}.$

A **critical number** for $f(x)$ is a number c so that either

(i) $f'(c) = 0$; or

(ii) $f'(c)$ does not exist.

A **critical point** is a point $(c, f(c))$ for a critical number c.

A **relative maximum** for a function $f(x)$ is a number c so that for some $a < c < b$ we have and all x so that $a < x < b$ we have $f(c) \geq f(x)$.

(A **relative minimum** is the same, with $f(c) \leq f(x)$...)

PICTURE

A point which is either a relative maximum or minimum is called a **relative extremum**. (The plural of 'extremum' is 'extrema'.)

A relative maximum or minimum can only be at a critical number.

The First Derivative Test for Relative Extrema

Suppose that c is a critical number for $f(x)$. Then c is a

relative maximum if $f'(x) > 0$ to the left of c and $f'(x) < 0$ to the right of c.

relative minimum if $f'(x) < 0$ to the left of c and $f'(x) > 0$ to the right of c.

not a relative extremum if $f'(x)$ has the same side on both sides of c.

Examples:

$f(x) = x^2$

$f(x) = -x^2$

$f(x) = x^3$

Sketching the graph of $f(x)$

(1) Determine the domain of $f(x)$

(2) Find $f'(x)$ and find the critical numbers. Determine the intervals of increase and decrease for $f(x)$.

(3) For each critical number c, plot the critical point $(c, f(c))$, and note whether each one is a relative maximum, relative minimum, or neither.

(4) Find x- and y- intercepts (if possible).

(5) Join up the critical points and the intercepts.

[NOTE: We're ignoring some things here – like asymptotes, and points of inflection, and concavity. We'll get to most of these later...]

Example(s)

Sketch the graph of the function from before.