The Derivative

Let $f(x)$ be a function and suppose that c is in the domain of f. The **derivative of f at c** is

$$\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

(supposing that this limits exists).

The function $f(x)$ is **differentiable at $x = c$** if $f'(c)$ exists.

The **derivative of $f(x)$** (as a function of x) is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

[The domain of this function is the set of values for which this limit exists.]

Things the derivative can mean:

[More details on all of these later]

- The slope of the tangent line. [The tangent line at $x = c$, in turn, is the best linear approximation to f at the point $(c, f(c))$.]

- The instantaneous rate of change of f.

- If $f(x)$ measures where a point is, $f'(x)$ measures the **velocity** of the function.

[The derivative also allows us to calculate maximum (or minimum) values of $f(x)$.]

Examples:

Calculate $f'(x)$ where:

1. $f(x) = 3x + 5$;

2. $f(x) = 2x^2 - 5x + 1$;

3. $f(x) = x^3$;

4. $f(x) = x^4$.

Let $f(x)$ be a function.

The **tangent line** to the curve $y = f(x)$ at a point $x = c$ is the straight line which

• Touches the curve $y = f(x)$ at $(c, f(c))$; and

• Is the straight line which best approximates the curve $y = f(x)$ and goes through $(c, f(c))$.

The tangent line to the graph of $y = f(x)$ at $x = c$.

The **slope** of the tangent line to $y = f(x)$ at $x = c$ is $f'(c)$.

The tangent line goes through the point $(c, f(c))$. So, we can use the point-slope form of a straight line to find the equation of the tangent line:

$$y - f(c) = f'(c)(x - c),$$

which means

$$y = f'(c)x + f(c) - f'(c)c.$$

Examples

Find the equations of the tangent line at $x = 1$ for the above examples.

Graph them...

A function $f(x)$ is **increasing** if when we increase x the function f increases too.

It is **decreasing** if when we increase x we *decrease* $f(x)$.

A straight line is **increasing** when the slope is **positive**. It is **decreasing** if the slope is **negative**.

We get a similar thing for derivatives...

Let $f(x)$ be a function, and suppose that $f'(c)$ exists. The function $f(x)$ is

- **increasing** at c if $f'(c) > 0$.

- **decreasing** at c if $f'(c) < 0$.

[PICTURES]

If $f(x)$ is differentiable at c then $f(x)$ is continuous at c.

The converse doesn't need to hold.

[There are continuous functions which aren't differentiable everywhere.]