From last time, we didn't deal with:

$$f_4(x) = x^4 - 24x^2 + 36x - 44.$$

(We want to find intervals of increase/decrease, relative extrema, intervals of concavity, points of inflection, intercepts, etc.)

Vertical asymptotes

A the (vertical) line $x = c$ is a vertical asymptote of the function $f(x)$ if

$$\lim_{x \to c^-} = \infty$$

(or is equal to $= -\infty$), and/or

$$\lim_{x \to c^+} = \infty$$

(or $-\infty$).

When we sketch the graph of a function with asymptotes, we usually include the asymptotes as dotted lines.

Determine vertical asymptotes in the following examples:

- $f_1(x) = \frac{1}{x-1}$
- $f_2(x) = \frac{x}{x^2-3x}$
- $f_3(x) = \frac{x^2+1}{\sqrt{x^2-1}}$

Horizontal asymptotes

The (horizontal) line $y = b$ is called a horizontal asymptote of the graph of $y = f(x)$ if

$$\lim_{x \longrightarrow \infty} f(x) = b$$

or

$$\lim_{x \longrightarrow -\infty} f(x) = b.$$

Horizontal asymptotes are also marked with dotted lines.

[So, now when we are graphing functions we should also include vertical and horizontal asymptotes, if any.]

Find horizontal asymptotes in the following examples

- $f_1(x) = \frac{3x-1}{x+2}$

- $f_2(x) = \frac{x^2-1}{1+2x-x^2}$

Note: In most simple-ish examples, vertical and horizontal asymptotes occur only for rational functions.

For a rational function:

A vertical asymptote occurs when the denominator is zero.

The horizontal asymptote will occur when the *degrees* of the numerator is the same as that of the denominator.

OK, so now let's sketch some functions.

$$f(x) = \frac{x}{\sqrt{1-x^2}}$$

$$f(x) = \frac{x^2+1}{x^2-1}$$

$$f(x) = x^{5/2}$$

$$f(x) = x^3 - 2x^2 + 3x - 6$$