Techniques of differentiation

[Another way of writing $f'(x)$ is as $\frac{df}{dx}$. This notation allows us to write things like

$$\frac{d}{dx}[x^2 - 3x + 1] = 2x - 3.$$

The power rule:

$$\frac{d}{dx}[x^n] = n.x^{n-1}.$$

(For any real number n.)

NOTE: This agrees with the calculations we made last time:

$$\frac{d}{dx}[x^3] = 3x^2,$$

and

$$\frac{d}{dx}[x^4] = 4x^3.$$

We can also use this to see:

$$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{\frac{1}{2}}] = \left(\frac{1}{2}\right) x^{-\frac{1}{2}}$$

and

$$\frac{d}{dx}[x^{\sqrt{2}}] = \left(\sqrt{2}\right) x^{\left(\sqrt{2}-1\right)}$$

and so on...

We have some other rules:

- If c is a constant and $f(x)$ is a differentiable function then:

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}\left[f(c)\right]$$

- (The Sum Rule) If $f(x)$ and $g(x)$ are differentiable then

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right].$$

- (The Product Rule) If $f(x)$ and $g(x)$ are differentiable then

$$\begin{aligned}\frac{d}{dx}\left[f(x).g(x)\right] &= \frac{d}{dx}\left[f(x)\right].g(x) + f(x)\frac{d}{dx}\left[g(x)\right]. \\ &= f'(x).g(x) + f(x).g'(x).\end{aligned}$$

- (The quotient rule) If $f(x)$ and $g(x)$ are differentiable then whenever $g(x) \neq 0$ we have

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

- **(The Chain Rule) If $f(y)$ and $g(x)$ are differentiable then**

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = g'(x).f'\left(g(x)\right).$$

Examples on board ...

Relative Rate of Change (and Percentage Rate of Change)

The **Relative Rate of Change** of a function $f(x)$ with respect to x is:

$$\frac{f'(x)}{f(x)}$$

The **Percentage Rate of Change** of $f(x)$ with respect to x is:

$$\frac{100f'(x)}{f(x)}.$$

Examples:

The population (in thousands) P of a city at time t after 1850 is given by

$$P(t) = 100 + 70t - 1.5t^2 + 0.01t^3.$$

What is the rate of change of the population in the year 2010?

What is the percentage rate of change of population with respect to time in 2010?