POP QUIZ!

- 1. What is the derivative of $f(x) = x^2 2x$?
- 2. What is the slope of the tangent line to $f(x) = x^2 2x$ at x = 2?

3. What is the equation of the tangent line to $f(x) = x^2 - 2x$ through the point (2, f(2))?

Relative Rate of Change (and Percentage Rate of Change)

The **Relative Rate of Change** of a function f(x) with respect to x is:

 $\frac{f'(x)}{f(x)}$

The Percentage Rate of Change of f(x) with respect to x is: $\frac{100f'(x)}{f(x)}.$

Examples:

The population (in thousands) P of a city at time t after 1850 is given by

$$P(t) = 100 + 70t - 1.5t^2 + 0.01t^3.$$

What is the rate of change of the population in the year 2010?

What is the percentage rate of change of population with respect to time in 2010?

Higher order derivatives

All this means is that we take a function f(x), differentiate it to get a new function f'(x), and then differentiate **it** to get yet another function, which we write as f''(x).

Notation/terminology If we differentiate a function f(x) n times, we get the n^{th} derivative, which we denote as

$$\frac{d^n f}{d^n x}$$
 or as $f^{(n)}(x)$.

For small values of n, we use a bunch of primes, so the second derivative is usually written



Example

Find the fourth derivative of $f(x) = 3x^3 + 4x^2 - 10x + 3$.

Find the second derivative of $f(x) = \frac{x}{x+1}$.

A natural interpretation of the second derivative is as follows:

Suppose that x represents time, and f(x) represents distance from the origin (along a line – the x-axis).

Then: The derivative f'(x) represents speed (positive if moving to the right), negative if moving to the left.

The second derivative $f''(x) = f^{(2)}$ represents acceleration (positive if f'(x) is increasing, negative if f'(x) is decreasing). More examples...