

Graded Homework 1: Due Wednesday September 13, 2017 at the beginning of class

For this homework, you may assume the existence of the set of integers, and the basic properties of addition, subtraction, multiplication and the order ('<') on them. You should be clear about when you use such properties (by saying something like 'By basic properties of arithmetic, ...'). You **may not** use rational numbers or decimal numbers.

When proving a numbered proposition, you may assume that any proposition with a smaller number has already been proved (whether or not that proposition appears on the homework or only on the worksheet).

1. (5 points) Prove Proposition 6:

Proposition 6. *For any integers a , b and c , if $a|b$ then $a|bc$.*

2. (5 points) Prove Proposition 12:

Proposition 12. *For any integers n , a and b , if $n | (a - b)$ then $n|(b - a)$.*

3. (5 points) Prove Proposition 13:

Proposition 13. *For any integers n , a , b and c , if $n | (a - b)$ and $n | (b - c)$ then $n | (a - c)$.*

4. Your friend has written the following proof of Proposition 8.

Proposition 8. *Let a , b and c be integers. If $a|b$ and $b|c$ then $a|c$.*

Proof. $a|b$ is $b = ak$. k is an integer. So $a|b = a|ak$, and $a|b$ is $a/b = k$ is an integer.

$b|c$ is $c = bk$, k an integer. So $b|c = b|bk$ and k is an integer.

If $a|c$ then $c = ak$. But $c = bk$ and $b = ak$ so c/a is an integer. Therefore $a|c$. \square

Professor Groves is not going to like it.

- (a) (5 points) Help your friend by writing **at least** two paragraphs of **at least** three sentences each explaining what Professor Groves is not going to like about their proof and the mistakes they have made (there are many things Professor Groves won't like, so don't just give one thing).
- (b) (5 points) Write your own (perfect) proof of Proposition 8.