

Graded Homework 2: Due Monday October 2, 2017 at the beginning of class

1. (10 points) For each of the following relations on a set X , determine which of the following properties it has: reflexive, symmetric and transitive, and explain why or why not. For those that are equivalence relations, describe the equivalence classes.
 - (i) For $X = \mathbb{Z}$, put $a \sim b$ when $a + b$ is even.
 - (ii) For $X = \mathbb{Z}$, put $a \sim b$ when $a + b$ is odd.
 - (iii) For $X = \mathbb{Z}$, put $a \sim b$ when $3 \mid (a + b)$.
 - (iv) For $X = \mathbb{Z}$, put $a \sim b$ when $a^2 = b^2$.
2. (5 points) For each of the following mathematical expressions, write down an expression that has exactly the opposite meaning. (The ‘negation’.)
 - (i) For all $\epsilon > 0$ there exists $\delta > 0$ so that for all x for which $|x - a| < \delta$ we have $|f(x) - f(a)| < \epsilon$.
 - (ii) For all integers n and m so that $n < m$ there is an integer k so that $n - k^2 = m + 1$.
 - (iii) For all natural numbers n and k and all integers a, b and c if $a + kb \equiv c \pmod{n}$ then $a - c \equiv kb \pmod{n}$.
3. (5 points) Decide whether Conjectures 19 and 20 are true or false. For each one, if it is true, prove it, and if it is false, prove that it is false (a counterexample would suffice in this case).
4. (5 points) Prove Lemma 30 from Worksheet 3.