1 Elementary number theory

We assume the existence of the natural numbers

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

and the integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\},$$

along with their most basic arithmetical and ordering properties. By this, we mean addition, subtraction and multiplication. Since division forces us to leave the world of integers, we will **not** assume that we can divide integers, and **fractions are not allowed** for now.

For example, we assume the truth of statements such as

"For all integers a and b, both $a + b$ and ab are integers."

"$0 < 1$"

"For all integers a and b we have $a + b = b + a$."

"For any pair of integers a and b, exactly one of the following is true: $a = b$, $a < b$ or $a > b$."

Definition 1. *Let a and b be integers. We say a* **divides** *b if there exists an integer k such that $ka = b$.*

Notation 2. *The notation $a|b$ means "a divides b".*

Proposition 3. *For any integers a, b and c, if $a|b$ and $a|c$ then $a|(b + c)$.*

Proposition 4. *For any integers a, b and c, if $a|b$ and $a|c$ then $a|(b - c)$.*

Conjecture 5. *For any integers a, b and c, if $a|(b + c)$, then $a|b$ or $a|c$.*

Proposition 6. *For any integers a, b and c, if $a|b$ then $a|bc$.*

Proposition 7. *For any integers a, b, d, x and y, if $d|a$ and $d|b$, then $d|(ax+by)$.*

Proposition 8. *For any integers a, b and c, if $a|b$ and $b|c$ then $a|c$.*

Proposition 9. $2 \nmid 1$.

The content of Proposition 9 should be interpreted as "2 does not divide 1".