

1 Elementary number theory, II

The setup is the same as in Worksheet I. You may assume all the Propositions from Worksheet I.

Proposition 10. *For any integer n we have $n \mid 0$.*

Proposition 11. *For any integers n and a we have $n \mid (a - a)$.*

Proposition 12. *For any integers n, a and b , if $n \mid (a - b)$ then $n \mid (b - a)$.*

Proposition 13. *For any integers n, a, b and c , if $n \mid (a - b)$ and $n \mid (b - c)$ then $n \mid (a - c)$.*

Definition 14. *Let a and b be integers and let n be a natural number. We say a is congruent to b modulo n if $n \mid (a - b)$.*

Notation 15. *We denote “ a is congruent to b modulo n ” by*

$$a \equiv b \pmod{n}.$$

Proposition 16. *For any integer a and natural number n , we have $a \equiv a \pmod{n}$.*

Proposition 17. *For any integers a and b and any natural number n , if $a \equiv b \pmod{n}$ then $b \equiv a \pmod{n}$.*

Proposition 18. *For any integers a, b and c and any natural number n , if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$.*

Conjecture 19. *For any integers a and b and any natural number n , if $a \equiv b \pmod{n}$ then $(-a) \equiv (-b) \pmod{n}$.*

Conjecture 20. *For any integer a and any natural number n , we have $a \equiv a^2 \pmod{n}$.*

Question 21. *What is the usual word for a number a so that $a \equiv 0 \pmod{2}$? What about those b so that $b \equiv 1 \pmod{2}$?*