Computer Algorithms I

Spring 2020

Final Exam Practice I

flow network with vertex capacities:

in addition, every vertex has a vertex capacity $\ell(v)$: total flow passing through v can be at most $\ell(v)$

how can the algorithms we learned be used to solve this problem?

idea: change the network so that a maximum flow in the new network corresponds to a maximum flow for the vertex capacity version - split every vertex v into v', v'' such that edge (v', v'') has capacity $\ell(v)$, edges entering v enter v' and edges leaving v leave from v''

Final Exam Practice II

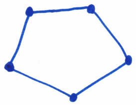
$VERTEX - COVER \in NP$

for graph $G = (V, E)$, a subset $V' \subseteq V$ is a vertex cover if for every edge $e = (u, v)$ of G, at least one of u, v is in V'

given a graph $G = (V, E)$ and a number k, does G have a vertex cover of size at most k?

$VERTEX - COVER = \{\langle G, k \rangle \ : \ G \text{ has vertex cover of size} \leq k\}$

Find a smallest vertex cover in the following graph:



Show that $VERTEX - COVER \in NP$.

hints: smallest vertex cover has size 3 (2 vertices can cover at most 4 edges), a certificate is a subset of vertices, verification algorithm checks whether a given vertex set is a vertex cover (i.e., every edge has an endpoint in the set)

Final Exam Practice III

$PRIME = \{m : m \text{ is a prime number}\}$

$COMPOSITE = \{m : m \text{ is a composite number}\}$

both problems are in P, but this is very difficult to show

for one of them it is not even easy to show that it is in NP

show that the other one is in NP!

note: numbers are written in binary, so input size is $\log m$, the number of bits

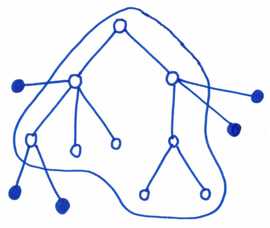
$COMPOSITE$ is in NP, a certificate is a divisor (different from 1 and m), and the verification algorithm checks if a claimed number divides m

Final Exam Problems IV

let $G = (V, E)$ be a undirected graph, and $V' \subseteq V$ be a subset of its vertices

does G have a spanning tree T such that every vertex $v \in V'$ is a leaf of T?

give an efficient algorithm for this problem



idea: there is such a spanning tree iff G restricted to $V - V'$ is connected, and every vertex in V' has a neighbor in $V - V'$