MCS 401 Spring 2020
Homework 2

February 10, 2020

1. Problem 1 [Exercise 5-2-1, page 122]:

   Exactly one hire occurs precisely when the first candidate is the best one. As the candidates arrive in random order, each candidate is equally likely to arrive first. Thus the probability of the best candidate coming first is $1/n$.

   Exactly n hires occur precisely when the candidates arrive in the order of decreasing rank, i.e., the worst candidate arrives first, the second worst arrives second, etc. There is just one ordering of the candidates for which this happens. As each of the $n!$ orderings are equally likely, the probability of this happening is $1/(n!)$.

2. Problem 2 [Problem 5-2-5, page 122]:

   Let X be the total number of inversions in the randomly selected permutation $A[1] \ldots A[n]$. We have that $X = \sum_{1 \leq i < j \leq n} X_{i,j}$, where

$$X_{i,j} = \begin{cases} 1 & if \text{A[i]} > \text{A[j]} \\ 0 & otherwise \end{cases}$$

   The expectation of each variable $X_{i,j}$ will be $\mathbb{E}(X_{i,j}) = 1.\frac{1}{2} + 0.\frac{1}{2} = \frac{1}{2}$. Then, using the linearity of expectation, we get:

$$\mathbb{E}(X) = \mathbb{E}(\sum_{i<j} X_{i,j}) = \binom{n}{2}.\frac{1}{2} = \frac{n(n-1)}{2}.\frac{1}{2} = \frac{n(n-1)}{4}.$$

3. Problem 3 [Exercise 5-3-3, page 128]:

   The total number of possible outputs of PERMUTE-WITH-ALL will be n^n, while the total number of n-permutations is $n!$. Each of the $n!$ permutations should be equally likely to occur as an output, but this is not true, because, for example, $3! = 6$ does not divide $3^3 = 27$.

4. Problem 4 [Exercise 6-3-1, page 159]:

   This would be the content of the array A at each step:

   STEP
   0 || 5 3 17 10 84 19 6 22 9
   1 || 5 3 17 22 84 19 6 10 9
   2 || 5 3 19 22 84 17 6 10 9

3 || 5 84 19 22 3 17 6 10 9
4 || 84 5 19 22 3 17 6 10 9
5 || 84 22 19 5 3 17 6 10 9
6 || 84 22 19 10 3 17 6 5 9

5. Problem 5 [Exercise 6-4-1, page 160]

   This would be the content of the array A at each step:

   STEP
   0 || 5 13 2 25 7 17 20 8 4
   1 || 5 13 20 25 7 17 2 8 4
   2 || 5 25 20 13 7 17 2 8 4
   3 || 25 5 20 13 7 17 2 8 4
   4 || 25 13 20 5 7 17 2 8 4
   5 || 25 13 20 8 7 17 2 5 4
   6 || 4 13 20 8 7 17 2 5 25
   7 || 20 13 4 8 7 17 2 5 25
   8 || 20 13 17 8 7 4 2 5 25
   9 || 5 13 17 8 7 4 2 20 25
   10 || 17 13 5 8 7 4 2 20 25
   11 || 2 13 5 8 7 4 17 20 25
   12 || 13 2 5 8 7 4 17 20 25
   13 || 13 8 5 2 7 4 17 20 25
   14 || 4 8 5 2 7 13 17 20 25
   15 || 8 4 5 2 7 13 17 20 25
   16 || 8 7 5 2 4 13 17 20 25
   17 || 4 7 5 2 8 13 17 20 25
   18 || 7 4 5 2 8 13 17 20 25
   19 || 2 4 5 7 8 13 17 20 25
   20 || 5 4 2 7 8 13 17 20 25
   21 || 2 4 5 7 8 13 17 20 25
   22 || 4 2 5 7 8 13 17 20 25
   23 || 2 4 5 7 8 13 17 20 25