MCS 401 Spring 2020
Homework 6

Exercise 24-3-1, Page 662

Figures 1 and 2 present Dijkstra's algorithm on the given graph with source s and z respectively. The d values are shown as they are updated on each iteration. The π values which indicate the parent of each vertex are shown in Table 1.

π values with s as source						
	1	2	3	4	5	6
s	NIL	NIL	NIL	NIL	NIL	NIL
t	NIL	s	s	s	s	s
x	NIL	NIL	t	t	t	t
y	NIL	s	s	s	s	s
z	NIL	NIL	NIL	y	y	y

π values with z as source						
	1	2	3	4	5	6
s	NIL	z	z	z	z	z
t	NIL	NIL	s	s	s	s
x	NIL	z	z	z	z	z
y	NIL	NIL	s	s	s	s
z	NIL	NIL	NIL	NIL	NIL	NIL

Table 1

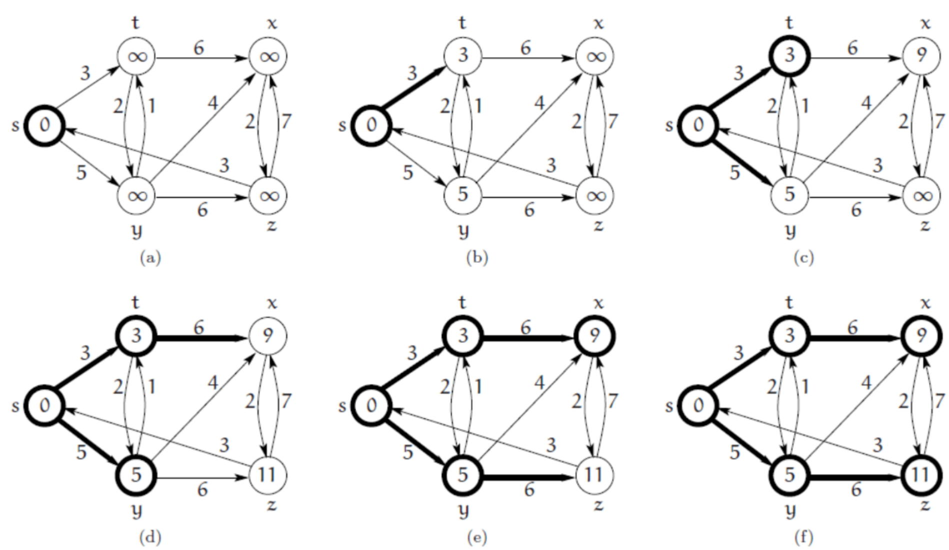

Figure 1: Dijkstra's algorithm with source s.

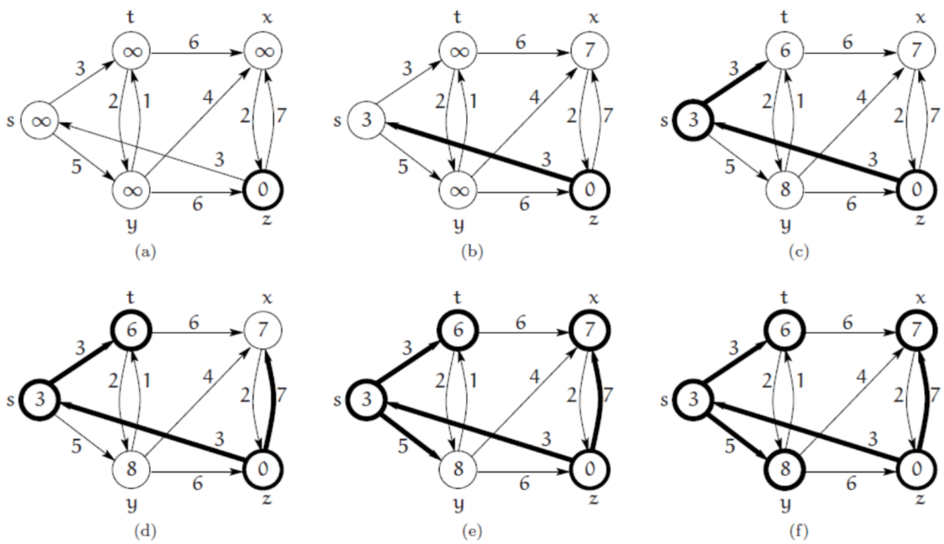

Figure 2: Dijkstra's algorithm with source z.

Exercise 26.2-3, Page 730

Figure 3 shows the residual networks if we execute Edmonds-Karp Algorithm on the network 26.1(a).

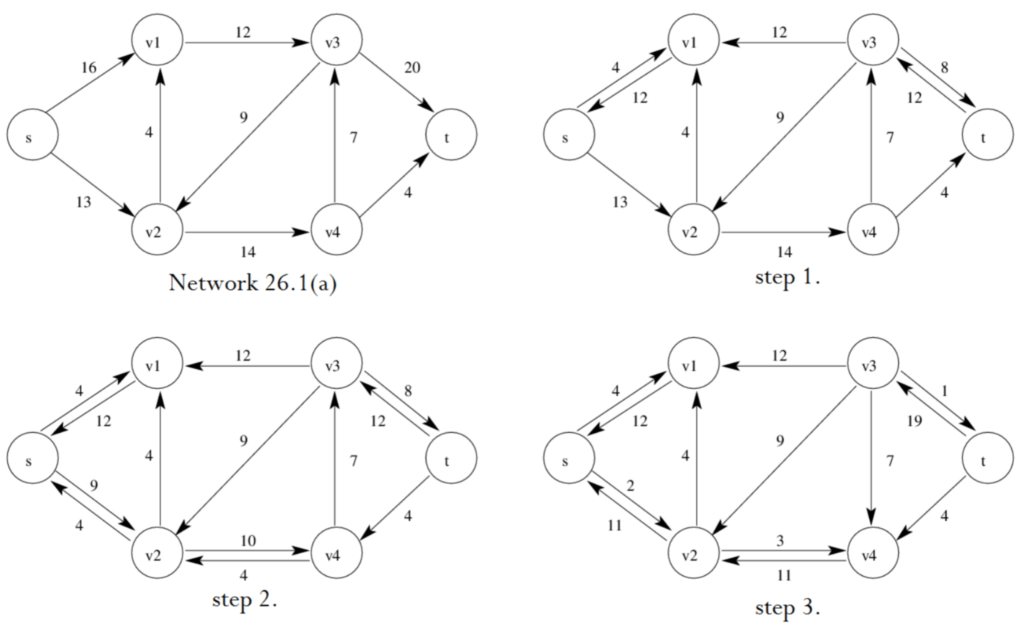

Figure 3: Residue networks generated by executing Edmonds-Karp Algorithm on Figure 26.1(a).

- In step 1, we improve by 12 along the path s–v_1–v_3–t;
- In step 2, we improve by 4 along the path s–v_2–v_4–t;
- In step 3, we improve by 7 along the path s–v_2–v_4–v_3–t.

After that no further improvement is possible, as there are no s–t paths in the residual network, and the maximum flow is $12 + 4 + 7 = 23$.

Exercise 34-2-1, Page 1065

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Let $|V_1| = |V_2| = n$; we enumerate vertices of G_1 and G_2 from 1 to n. Note that of $|V_1| \neq |V_2|$ then G_1 and G_2 are non-isomorphic to begin with hence we can immediately reject in this case. Consider a following certificate with a corresponding verification procedure. The certificate c is a permutation $\{i_1, \ldots, i_n\}$ of numbers $1 \ldots n$. For each vertex $v_k \in V_2$ we change the enumeration of vertices as $1 \to i_1, 2 \to i_2, \ldots, n \to i_n$ i.e $v_k \to v_{i_k}$. Now we check if the edge set of G_1 and the edge set of re-enumerated G_2 are the same. If they are, we accept. Otherwise, we reject.

The procedure above runs in polynomial time. By definition, G_1 and G_2 are isomorphic if and only if there is a permutation of V_2 which turns G_2 into G_1. Therefore, we can use permutation is a certificate which leads to acceptance. If G_1 and G_2 are non-isomorphic, no such permutation exists and the procedure never accepts.