Linear Quadratic Gaussian Poisson (LQGP) Problem with Multistage Manufacturing System Application[1]

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Overview

1. LQGP Problem.

2. Stochastic Dynamic Programming (SDP).

3. Multistage Manufacturing System (MMS): Computational Example.

4. Conclusions.

Abstract

The Linear Quadratic Gaussian Poisson (LQGP) problem denotes an optimal control problem with linear dynamics and quadratic costs with both Gaussian and Poisson noise disturbances (see Westman and Hanson [ACC97]).

A numerical example for a multistage manufacturing system (MMS) application of the LQGP problem is presented for the constrained control problem. The MMS is a *hybrid system*, a combination of discrete (e.g., Markov chains or Poisson Noise) and continuous systems in an uncertain environment.

Part 1. LQGP Problem

1.1. Defining Stochastic Differential Equation (SDE):

- **Linear (L) Dynamics with Gaussian (G) and Poisson (P) Noise:**

$$\begin{aligned} d\mathbf{X}(t) &= [A(t)\mathbf{X}(t) + B(t)\mathbf{U}(t) + \mathbf{C}(t)]dt \\ &+ G(t)d\mathbf{W}(t) + [H_1(t)\mathbf{X}(t)]d\mathbf{P}_1(t) \\ &+ [H_2(t)\mathbf{U}(t)]d\mathbf{P}_2(t) + H_3(t)d\mathbf{P}_3(t) \end{aligned}$$

- **Initial Conditions:** $\mathbf{X}(t_0) = \mathbf{x_0}$; $t_0 < t < t_f$

- **State vector:** $\mathbf{X} = [X_i]_{m\times 1}$;

- **Control Vector:** $\mathbf{U} = [U_i(\mathbf{X}(t), t)]_{n\times 1}$;

1.2. SDE Coefficients:

- **Linear Plant Vector:**

$$\mathbf{F}(\mathbf{x},\mathbf{u},t) = [A(t)\mathbf{X}(t) + B(t)\mathbf{U}(t) + \mathbf{C}(t)];$$

with time-varying arrays:

$$\begin{aligned} A(t) &= [A_{ij}(t)]_{m\times m}, \\ B(t) &= [B_{ij}(t)]_{m\times n}, \\ C(t) &= [C_i(t)]_{m\times 1}\ ; \end{aligned}$$

- **Gaussian Noise Amplitude Array:**

$$G(t) = [G_{ij}(t)]_{m\times r}\ ;$$

- **Poisson Jump Amplitude Coefficient Arrays:**

$$\begin{aligned} [H_1(t)\mathbf{x}] &= [\sum_k H_{1ijk}(t)x_k]_{m\times q_1}, \\ [H_2(t)\mathbf{u}] &= [\sum_k H_{2ijk}(t)u_k]_{m\times q_2}, \\ H_3(t) &= [H_{3ij}(t)]_{m\times q_3}. \end{aligned}$$

1.3. Infinitesimal Moments for Noise Processes:

- **Poisson Jumps and Moments:**

$$\mathbf{P}_\ell(t^+_{\ell,i,j}) - \mathbf{P}_\ell(t^-_{\ell,i,j}) = \eta_{\ell,i,j},$$

provided there exists jump $t_{\ell,i,j}$ of the $d\mathbf{P}_\ell$ process;

$$\begin{aligned}\text{Mean}[d\mathbf{P}_\ell(t)] &= \Lambda_\ell(t)dt\left[\int_{\mathcal{Z}_{\ell,i}} z_i\phi_{\ell,i}dz_i\right]_{q_\ell\times 1}\\ &\equiv \Lambda_\ell\overline{\mathbf{Z}}_\ell dt\end{aligned}$$

$$\begin{aligned}\text{Covar}[d\mathbf{P}_\ell(t)] &= \Lambda_\ell(t)dt\left[\int_{\mathcal{Z}_{\ell,i}} (z_i - \overline{Z}_{\ell,i})^2\phi_{\ell,i}dz_i\right]_{q_\ell\times 1}\\ &\equiv \Lambda_\ell(t)\sigma_\ell(t)dt = [\lambda_{\ell,i}\sigma_{\ell,i}dt]_{q_\ell\times 1}\end{aligned}$$

where

- $\Lambda_\ell(t) = Diag(\lambda_\ell(t)) = [\lambda_{\ell,i}(t)\delta_{i,j}]$,
- $\lambda_\ell(t)$ = Poisson Jump/Sojourn Rate,
- $\phi^T_\ell(\mathbf{z}, t)$ = Density of Vector Amplitude Marks,
- $\overline{\mathbf{Z}}_\ell$ = Mean of Vector Amplitude Marks,
- σ_ℓ = Variance of Vector Amplitude Marks;

- **Gaussian and Mixed Moments:**

$$\text{Mean}[d\mathbf{W}(t)] = 0_{r\times r},$$

$$\text{Covar}[d\mathbf{W}(t), d\mathbf{W}^T(t)] = I_r dt,$$

$$\text{Covar}[d\mathbf{W}(t), d\mathbf{P}^T_\ell(t)] = 0_{r\times q_\ell};$$

1.4. Continued with State Moments:

- **State Jumps and Conditional Moments:**

$$[\mathbf{X}](t_{\ell,i,j}) = \left\{ \begin{array}{cc} [[H_1(t_{\ell,i,j})\mathbf{X}(t^-_{\ell,i,j})]_i\eta_{\ell,i,j}]_{m\times 1}, & \ell = 1 \\ [[H_2(t_{\ell,i,j})\mathbf{U}(t^-_{\ell,i,j})]_i\eta_{\ell,i,j}]_{m\times 1}, & \ell = 2 \\ [[H_3(t_{\ell,i,j})]_i\eta_{\ell,i,j}]_{m\times 1}, & \ell = 3 \end{array} \right\},$$

provided there exists jump $t_{\ell,i,j}$ of the $d\mathbf{P}_\ell$ process;

$$\begin{aligned} \text{Mean}[d\mathbf{X}(t) \quad &| \quad \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \\ &= \quad [A(t)\mathbf{x} + B(t)\mathbf{u} + \mathbf{C}(t) + [H_1(t)\mathbf{x}]\Lambda_1(t)\overline{\mathbf{Z}}_1 \\ &+ \quad [H_2(t)\mathbf{u}]\Lambda_2(t)\overline{\mathbf{Z}}_2 + H_3(t)\Lambda_3(t)\overline{\mathbf{Z}}_3]\, dt; \end{aligned}$$

$$\begin{aligned} \text{Covar}[d\mathbf{X}(t) \quad &| \quad \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \\ &= \quad \Big[(GG^T)(t) + [H_1(t)\mathbf{x}]\Lambda_1(t)\sigma_1(t)[H_1(t)\mathbf{x}]^T \\ &+ \quad [H_2(t)\mathbf{u}]\Lambda_2(t)\sigma_2(t)[H_2(t)\mathbf{u}]^T \\ &+ \quad H_3(t)\Lambda_3(t)\sigma_3(t)H_3^T(t)\Big]\, dt. \end{aligned}$$

1.5. Quadratic (Q) Performance Index:

$$
\begin{aligned}
V(\mathbf{X},\mathbf{U},t) &= \frac{1}{2}\mathbf{X}^T(t_f)S(t_f)\mathbf{X}(t_f) \\
&+ \frac{1}{2}\int_t^{t_f}\left[\mathbf{X}^T(\tau)Q(\tau)\mathbf{X}(\tau)+\mathbf{U}^T(\tau)R(\tau)\mathbf{U}(\tau)\right]d\tau,
\end{aligned}
$$

where

- t_f = Time Horizon $(t_f \geq t)$,
- $S = [S_{i,j}]_{m\times m}$ = Quadratic Salvage Cost Matrix (positive semi-definite),
- $Q = [Q_{i,j}]_{m\times m}$ = Quadratic State Cost Matrix (positive semi-definite),
- $R = [R_{i,j}]_{n\times n}$ = Quadratic Control Cost Matrix (positive definite),
- $V(\mathbf{X}(t_f),\mathbf{U}(t_f),t_f) = \frac{1}{2}\mathbf{X}^T(t_f)S(t_f)\mathbf{X}(t_f)$ = Final Condition for Salvage Cost.

Part 2. Stochastic Dynamic Programming

2.1. Stochastic Dynamic Programming Formulation:

- Optimal, Expected Performance:

$$v(\mathbf{x},t) \equiv \min_{\mathbf{u}(t,t_f]} \left[\underset{\{\mathbf{P},\mathbf{W}\}(t,t_f]}{\mathrm{Mean}} \left[V(\mathbf{X},\mathbf{U},t) \mid \mathbf{X}(t)=\mathbf{x}, \mathbf{U}(t)=\mathbf{u} \right] \right];$$

- PDE of Stochastic Dynamic Programming

$$\begin{aligned} 0 \;&=\; \frac{\partial v}{\partial t}(\mathbf{x},t) + \min_{\mathbf{u}} \Big[\left(A(t)\mathbf{x} + B(t)\mathbf{u} + \mathbf{C}(t)\right)^T \nabla_x v(\mathbf{x},t) \\ &+\; \frac{1}{2}\left(GG^T\right)(t) : \nabla_x \nabla_x^T v(\mathbf{x},t) + \frac{1}{2}\mathbf{x}^T Q(t)\mathbf{x} + \frac{1}{2}\mathbf{u}^T R(t)\mathbf{u} \\ &+\; \Sigma_i \lambda_{1,i} \int_{\mathcal{Z}_{1i}} [v(\mathbf{x} + [H_1\mathbf{x}]_i z_i, t) - v(\mathbf{x},t)]\, \phi_{1,i}(z_i,t) dz_i \\ &+\; \Sigma_i \lambda_{2,i} \int_{\mathcal{Z}_{2i}} [v(\mathbf{x} + [H_2\mathbf{u}]_i z_i, t) - v(\mathbf{x},t)]\, \phi_{2,i}(z_i,t) dz_i \\ &+\; \Sigma_i \lambda_{3,i} \int_{\mathcal{Z}_{3i}} [v(\mathbf{x} + [H_3]_i z_i, t) - v(\mathbf{x},t)]\, \phi_{3,i}(z_i,t) dz_i \Big], \end{aligned}$$

  - subject to the final condition $v(\mathbf{x},t_f) = \frac{1}{2}\mathbf{x}^T S(t_f)\mathbf{x}$,
  - where $A : B = \Sigma_i \Sigma_j A_{i,j} B_{j,i} = \mathrm{Trace}[AB^T]$.

2.2. LQGP Ansatz and Results:

- LQGP Assumption for Optimal, Expected Value:

$$\begin{aligned} v(\mathbf{x},t) &= \frac{1}{2}\mathbf{x}^T S(t)\mathbf{x} + \mathbf{D}^T(t)\mathbf{x} + E(t) \\ &+ \frac{1}{2}\int_t^{t_f} \left(GG^T\right)(\tau) : S(\tau)d\tau, \end{aligned}$$

  where $S(t)$, $\mathbf{D}^T(t)$ and $E(t)$ are coefficients to be determined, with final conditions

  - $S(t_f)$ specified by final salvage costs, recycled for $t < t_f$,
  - $\mathbf{D}(t_f) = \mathbf{0}_{m\times 1}$,
  - $E(t_f) = 0$;

- Regular or Unconstrained Optimal Control:

$$\mathbf{U}_{\mathrm{reg}}(t) = -\widehat{R}^{-1}(t)\widehat{B}^T(t)\left[S(t)\mathbf{x} + \mathbf{D}(t)\right];$$

- Constrained Optimal Control:

$$U_i^*(t) = \min[U_{\max,i}(t), \max[U_{\min,i}(t), U_{\mathrm{reg},i}(t)]],$$

  for example of Component-Wise or Hypercube Constraints:

$$U_{\min,i}(t) \le u_i(t) \le U_{\max,i}(t);$$

2.3. LQGP Coefficient Equations:

- Quadratic Coefficient $S(t)$ Nonlinear Equation:

$$\begin{aligned}
0_{m\times m} &= \dot{S}(t) + \left[A^T S + SA + Q\right](t) \\
&+ \tilde{\Gamma}_1(t) - \left[S\widehat{B}\widehat{R}^{-1}\widehat{B}^T S\right](t) \\
&\equiv \dot{S}(t) - \mathcal{S}(S(t),t);
\end{aligned}$$

- Linear Coefficient $\mathbf{D}(t)$ Equation:

$$\begin{aligned}
\mathbf{0}_{m\times 1} &= \dot{\mathbf{D}}(t) + \left[\left(A + (\Lambda_1\overline{\mathbf{Z}}_1)^T H_1^T\right)^T \mathbf{D}\right](t) \\
&+ \left[S\left(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3\right) - S\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t) \\
&\equiv \dot{\mathbf{D}}(t) - \mathcal{D}(\mathbf{D}(t), S(t), t);
\end{aligned}$$

- Scalar Coefficient $E(t)$ Equation:

$$\begin{aligned}
0 &= \dot{E}(t) + \left[(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3)^T\,\mathbf{D}\right](t) \\
&+ \frac{1}{2}\left[\left(H_3^T S H_3\right) : \Lambda_3\overline{ZZ}_3 - \mathbf{D}^T\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t) \\
&\equiv \dot{E}(t) - \mathcal{E}(\mathbf{D}(t), S(t), t);
\end{aligned}$$

2.4. LQGP Coefficient Definitions:

where

- $$\Gamma_1(t) \equiv \left[\left([H_1^T]_i S[H_1]_j : \Lambda_1 \overline{ZZ}_1\right)(t)\right]_{m\times m} + 2\left[(\Lambda_1 \overline{\mathbf{Z}}_1)^T H_1^T S(t)\right]_{m\times m},$$

- $$\Gamma_2(t) \equiv \left[\left([H_2^T]_i S[H_2]_j : \Lambda_2 \overline{ZZ}_2\right)(t)\right]_{n\times n},$$

- $$\overline{ZZ}_\ell(t) \equiv \overline{\sigma}_\ell(t) + \left(\overline{\mathbf{Z}}_\ell \overline{\mathbf{Z}}_\ell^T\right)(t) = \left[\sigma_{\ell,i}\delta_{i,j} + \overline{Z}_{\ell,i}\overline{Z}_{\ell,j}\right]_{q_\ell \times q_\ell}$$
  for $\ell = 1$ to 3,

- $$\widehat{R}(t) \equiv R(t) + \tilde{\Gamma}_2(t),$$

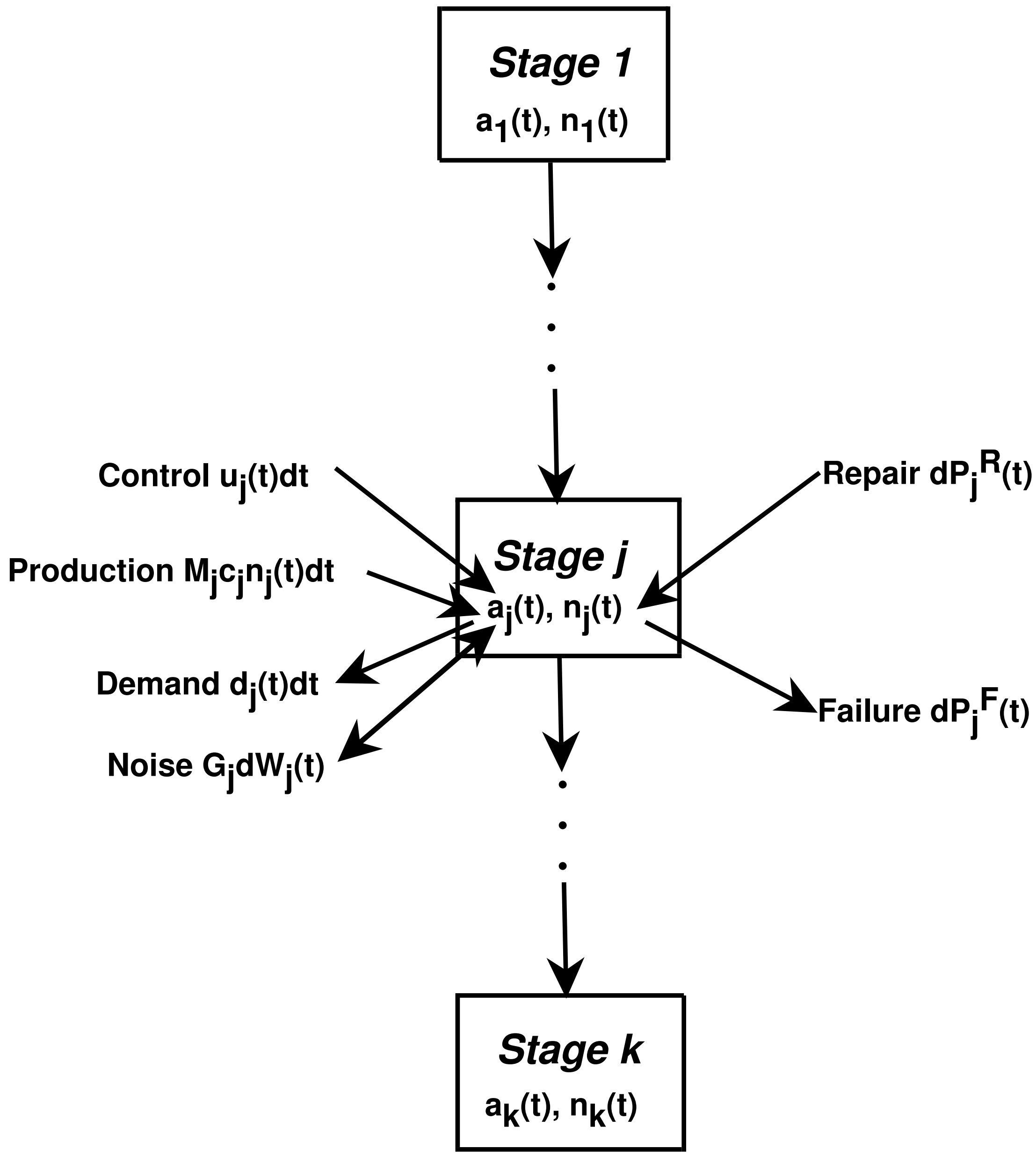
- $$\widehat{B}(t) \equiv B(t) + ((\Lambda_2 \overline{\mathbf{Z}}_2)^T H_2^T)(t),$$

- $$\tilde{\Gamma}_\ell = \Gamma_\ell + \Gamma_\ell^T.$$

Part 3. Stochastic Dynamic Programming

3.1. Block Diagram for MMS:

Mutlistage Manufacturing System



3.2. Model Assumptions for MMS:

- Loading (input of raw materials) and Unloading (delivery of finished goods) stages are not considered part of the MMS.

- MMS consists of **k** stages.

- Stage j is Preceded by Stage $j-1$ and followed by stage $j+1$.

- For Stage j, the State Variables are the Surplus Aggregate Level, $a_j(t)$, and the Number of Operational Workstations, $n_j(t)$.

3.3. State Variables:

- Surplus Aggregate Level for Stage j, $a_j(t)$, is Number of Pieces successfully completing j stages of the Manufacturing Process that are not defective and not consumed by stage $j+1$.

- For each stage, the Workstations are assumed to be Identical and Produce at Same Rate.

- Number of Operational Workstations, $n_j(t)$, at Stage j is constrained by $0 \leq n_j(t) \leq N_j(t)$, where $N_j(t)$ is Total Number of Workstations.

- For every Stage j, one or more Workstations may randomly be Repaired ($\Delta n_j(t) \geq +1$) or Fail ($\Delta n_j(t) \leq -1$).

- Production Rate, $c_j(t)$, is Utilization (Fraction of Time Busy) for each Workstation on each Stage and is constrained by $0 \leq c_j(t) \leq c_{\max,j}(t)$, where,

  $$c_{\max,j}(t) = \min[1, c_{j-1}(t) * M_{j-1} * n_{j-1}(t)/(M_j * n_j(t))],$$

  due to the physical and production limitations, respectively.

3.4. MMS Stochastic Differential Equation:

$$
\begin{aligned}
d\mathbf{X}(t) &= \begin{bmatrix} d\mathbf{a}(t) \\ d\mathbf{n}(t) \end{bmatrix} \\
&= \begin{bmatrix} [\mathrm{Diag}[\mathbf{M}]\mathrm{Diag}[\mathbf{c}(t)]\mathbf{n}(t) + \mathbf{u}(t) - \mathbf{d}(t)]\, dt + \mathrm{Diag}[\mathbf{g}(t)]d\mathbf{W}(t) \\ \left[d\mathbf{P}^R(t) - d\mathbf{P}^F(t)\right] \end{bmatrix}
\end{aligned}
$$

where

- $\mathrm{Diag}[\mathbf{y}(t)] = [\Sigma_j \delta_{i,j} y_j]_{k\times k}$ = Diagonalization of Vector with components $y_i(t)$ for $i = 1$ to k,
- $\mathbf{a}(t)$ = Surplus Aggregate Level Vector ($k \times 1$ State Vector Component): Production Surplus or Shortfall,
- $\mathbf{n}(t)$ = Number of Operational Workstations Vector ($k \times 1$ State Vector Component),
- $\mathbf{M}$ = Maximum Production Capacity Vector ($k \times 1$),
- $\mathbf{c}(t)$ = Production Rate Vector ($k \times 1$): $c_j(t)$ = Utilization of each Workstation for Stage j at time t,
- $\mathbf{u}(t)$ = Control Vector ($k \times 1$),
- $\mathbf{d}(t)$ = Demand Rate Vector ($k \times 1$),
- $\mathbf{g}(t)$ = Gaussian Noise Coefficient Vector ($k \times 1$),
- $d\mathbf{W}(\mathbf{t})$ = Gaussian Noise ($k \times 1$),
- $d\mathbf{P}^R(t)$ = Repair Process Poisson Noise Vector ($k \times 1$) for Broken Workstations,
- $d\mathbf{P}^F(t)$ = Failure Process Poisson Noise Vector ($k \times 1$) for Operational Workstations.

3.5. MMS Performance Index:

$$V(\mathbf{X}(t), \mathbf{u}(t), t) = \frac{1}{2}\mathbf{X}^T(t_f)S(t_f)\mathbf{X}(t_f) + \frac{1}{2}\int_t^{t_f} \mathbf{u}^T(\tau)R(\tau)\mathbf{u}(\tau)d\tau$$

where

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{a}(t) \\ \\ \mathbf{n}(t) \end{bmatrix}$$

- Performance Index to be Minimized is a Quadratic Cost-To-Go Form,

- Salvage or Terminal Cost Matrix S is a symmetric, positive semi-definite $2k \times 2k$ matrix,

- Salvage Cost is based on *Just In Time* or *Stockless Production* (see Hall [6]). Penalty is imposed if there is Surplus or Shortfall of production at end of planning horizon in any stage of manufacturing system,

- Instantaneous Quadratic Cost Control Coefficient $R(t)$ is a symmetric, positive definite $k \times k$ matrix, so that integral term in performance index is quadratic in control effort.

3.6. MMS 3 Stage Numerical Example:

- MMS with $k = 3$ Stages,
- Planning Horizon $T = t_f = 40$ hours,
- Surplus Aggregate Level is Initially $\mathbf{a}(t) = \mathbf{0}_3$,
- Fixed Demand Rate $\mathbf{d}(t) = [285 \quad 285 \quad 285]^T$ pieces,
- No Gaussian Random Fluctuations $\mathbf{G}(t) = \mathbf{0}_3$,
- Total Number of Workstations $\mathbf{N}(t) = [3 \quad 5 \quad 4]^T$,
- Production Capacity $\mathbf{M}(t) = [117 \quad 71 \quad 88]^T$ in pieces per hour,
- Mean Time to Repair $1/\lambda^R(t) = \text{Diag}[[2.5 \quad 4.0 \quad 1.5]^T]$, in hours,
- Mean Time between Failures $1/\lambda^F(t) = \text{Diag}[[25.0 \quad 21.5 \quad 30.0]^T]$, in hours,
- Salvage Cost Coefficient Matrix:

$$S(t_f) = \begin{bmatrix} S_1(t_f) & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} \text{ where } S_1(t_f) = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.9 & 0 \\ 0 & 0 & 2.6 \end{bmatrix}$$

- Quadratic Cost Coefficient:

$$R(t) = \begin{bmatrix} 1.3E4 & 0 & 0 \\ 0 & 0.8E4 & 0 \\ 0 & 0 & 0.6E4 \end{bmatrix}.$$

3.7. MMS 3 Stage Numerical Results:

- Production Rates are the utilization of all operational workstations for a given stage,

- Virtual Static Production Rate is Rate of production not influenced by Repairs or Failures in remaining time of Planning Horizon (t_f),

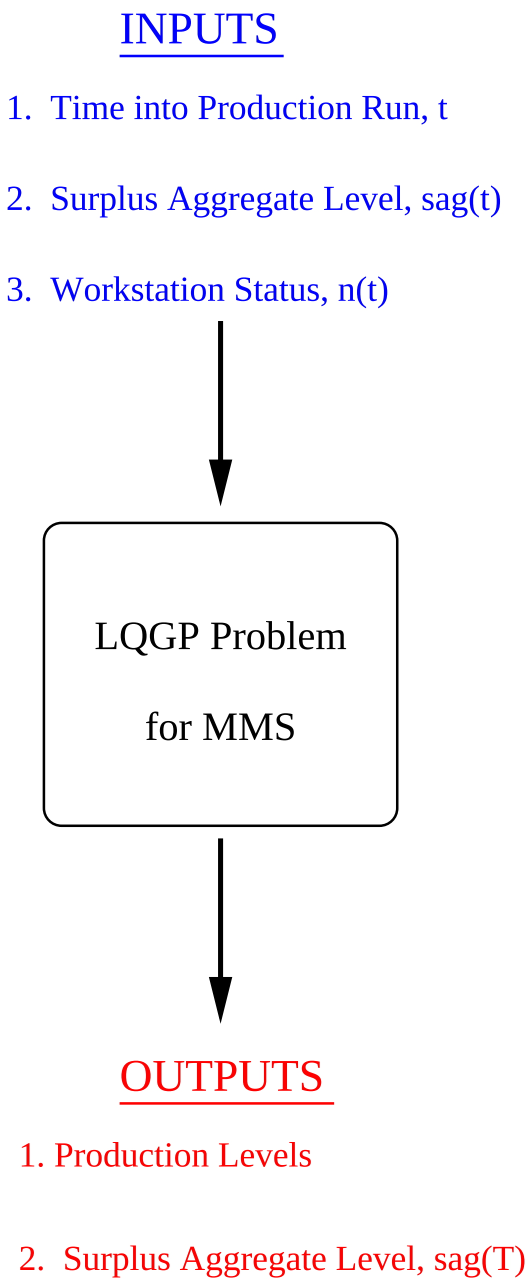
- Regular and Constrained Control Production Rates anticipate repairs and failures in the remaining time of the planning horizon,

- Time is measured from the beginning of the planning horizon.

3.8. MMS User Guide for Production Rate Graphs:

- Determine Time T now into Planning Horizon $T \le t_f$,
- Determine Number of Operational Workstations $\mathbf{n}(t)$ for all stages at time T,
- Set Production Rate to Constrained Control Production Rate,
- Determine Projected Surplus Aggregate Level.

3.9. MMS User Guide for Production Rate Graphs:



3.10. Continued: Figure 1.

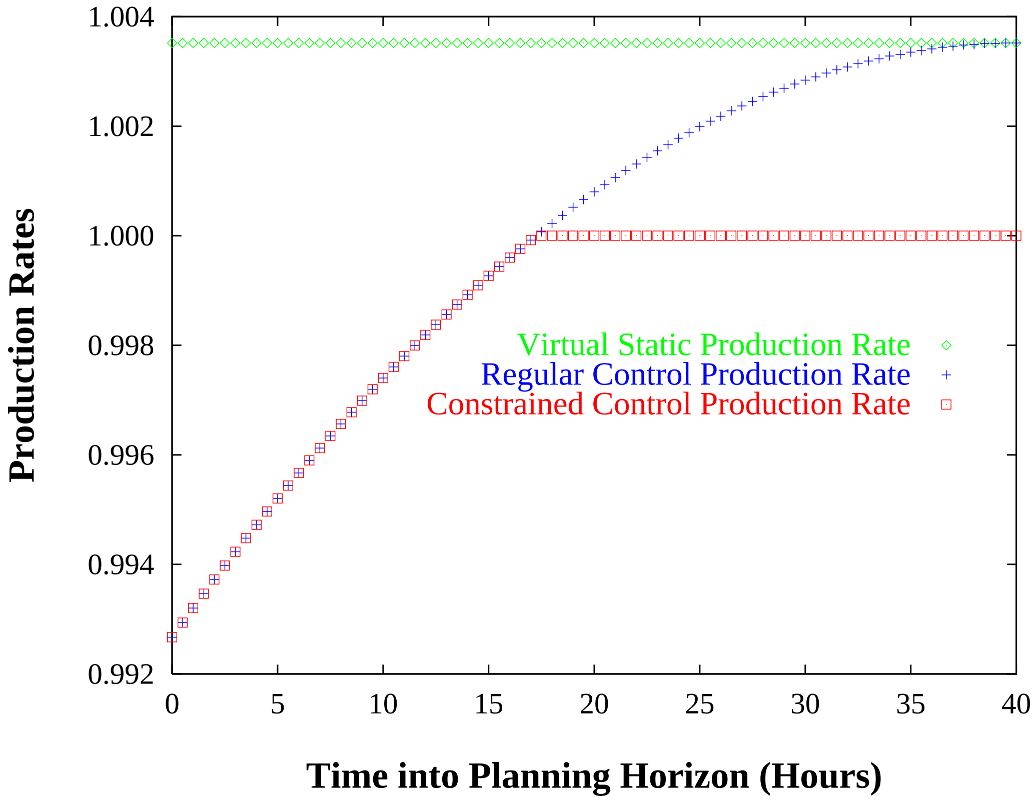


Figure 1: Production Rates for stage 2 with [3,4,*] active workstations.

- **Constrained Production Rate saturates at physical limitation of workstations**
- **Regular and Constrained Production Rates anticipate workstation repair and failure**
- **Surplus Aggregate Level becomes negative due to shortfall of production – see next slide**

3.11. Continued: Figure 2.

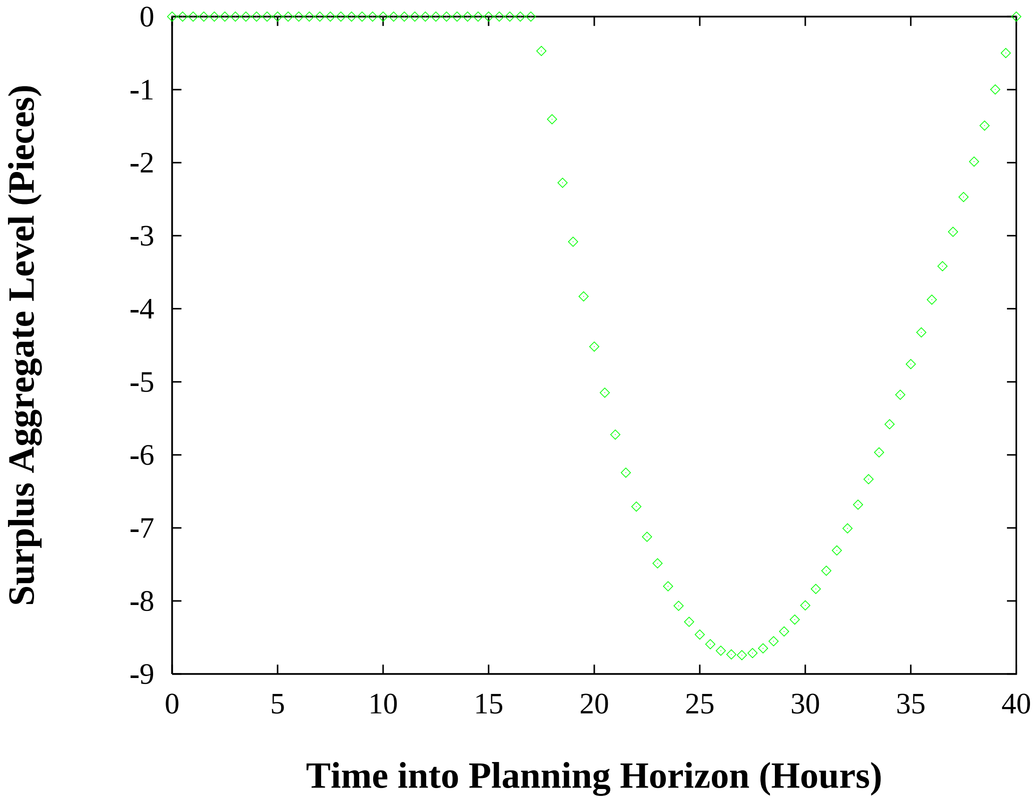


Figure 2: Surplus Aggregate Level for stage 2 with [3,4,*] active workstations.

- **Negative Values for Surplus Aggregate Level indicates a Shortfall of production.**

3.12. Continued: Figures 3 and 4.

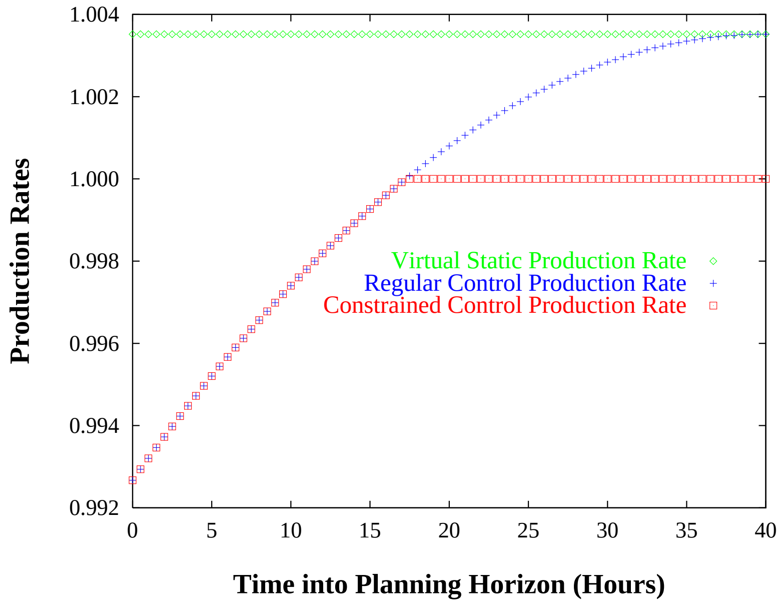


Figure 3: Production Rates for stage 2 with [3,4,*] active workstations.

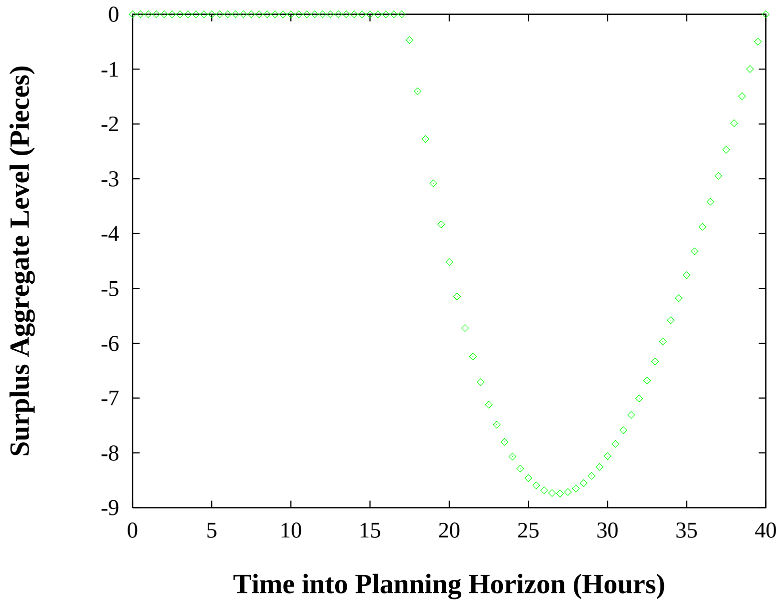


Figure 4: Surplus Aggregate Level for stage 2 with [3,4,*] active workstations.

3.13. Continued: Figure 5.

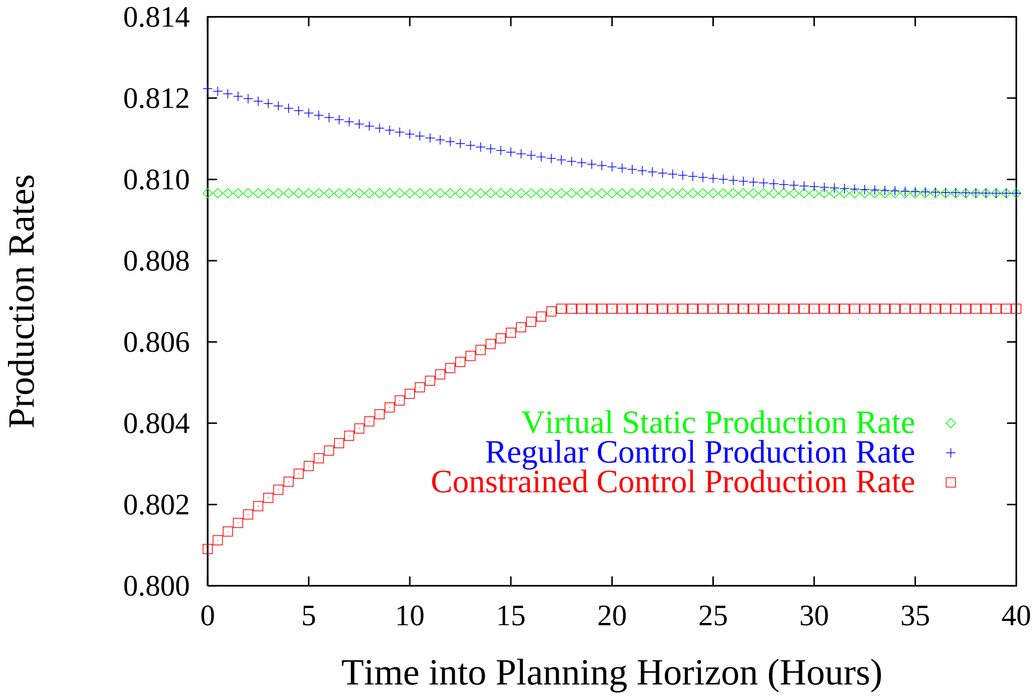


Figure 5: Production Rates for stage 3 with [3,4,4] active workstations.

- **Constrained Production Rate saturates due to shortfall of production from stage 2 – see next slide**
- **Regular and Constrained Production Rates anticipate workstation failure ONLY**
- **Surplus Aggregate Level becomes negative due to shortfall of production**

3.14. Continued: Figures 6 and 7.

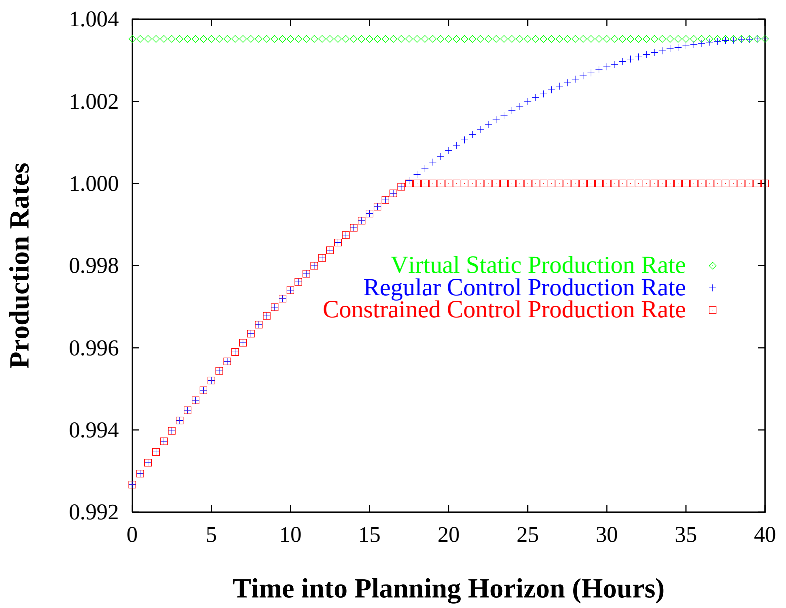


Figure 6: Production Rates for stage 2 with [3,4,*] active workstations.

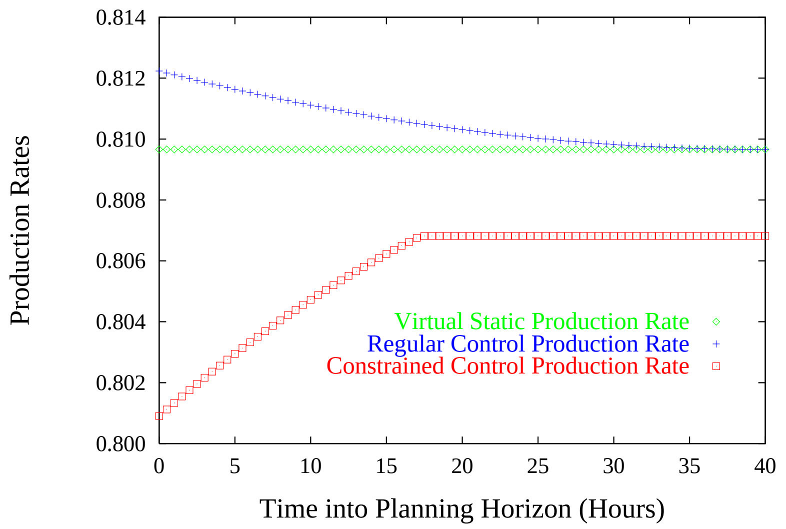


Figure 7: Production Rates for stage 3 with [3,4,4] active workstations.

3.15. Continued: Figures 8 and 9.

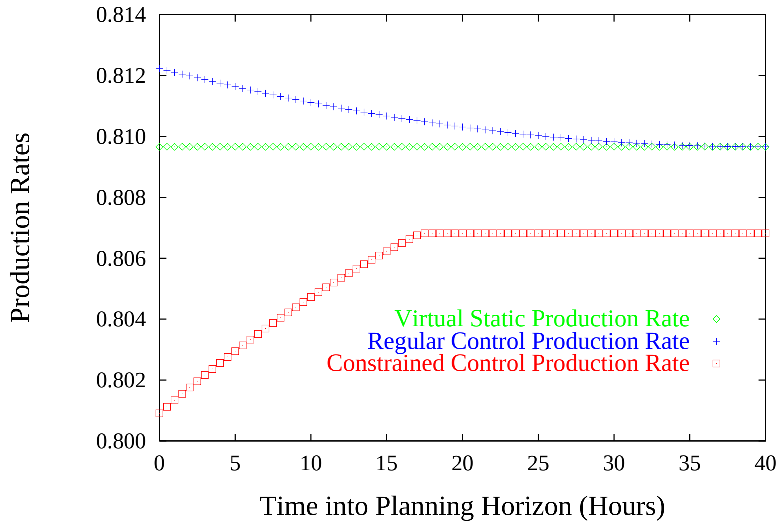


Figure 8: Production Rates for stage 3 with [3,4,4] active workstations.

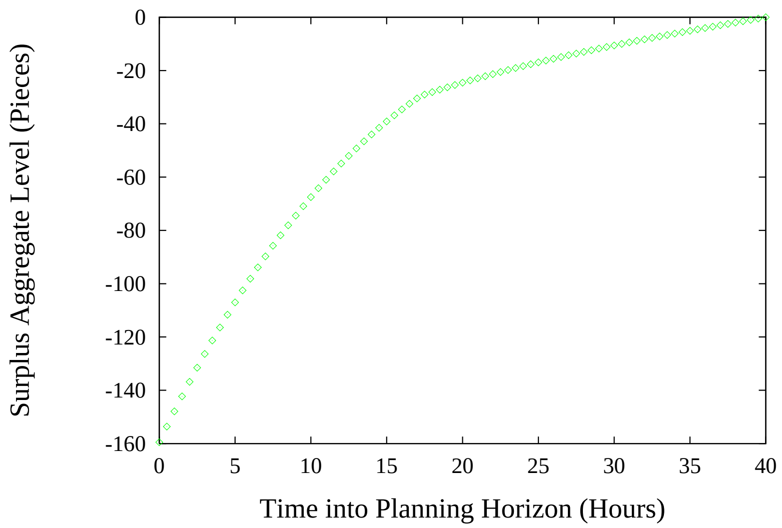


Figure 9: Surplus Aggregate Level for stage 3 with [3,4,4] active workstations.

3.16. Continued: Figure 10.

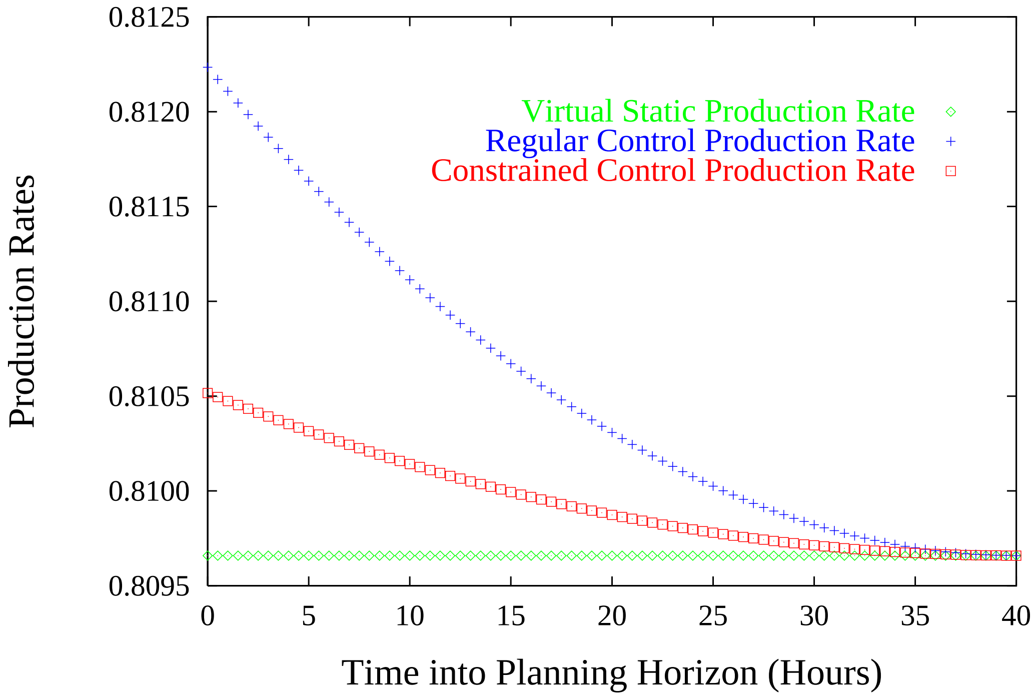


Figure 10: Production Rates for stage 3 with [3,5,4] active workstations.

- **Constrained Production Rate saturates due to shortfall of production from stage 2,**
- **Regular and Constrained Production Rates anticipate workstation failure ONLY.**
- **Surplus Aggregate Level becomes negative due to shortfall of production.**

3.17. Continued: Figures 11 and 12.

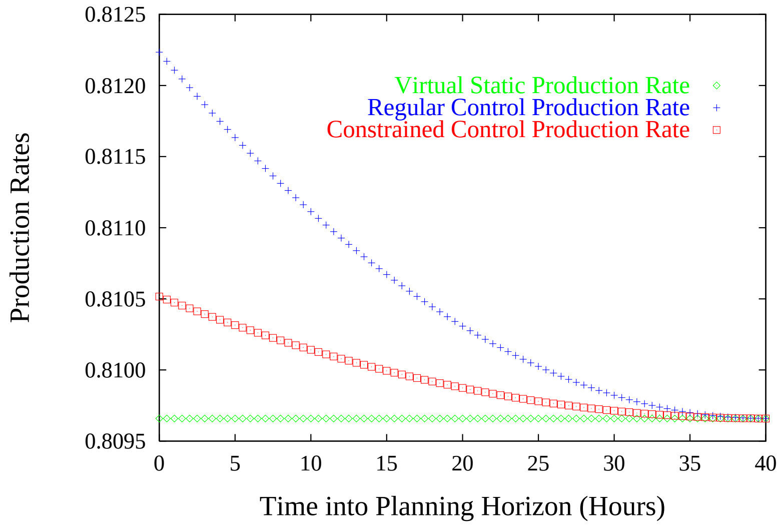


Figure 11: Production Rates for stage 3 with [3,5,4] active workstations.

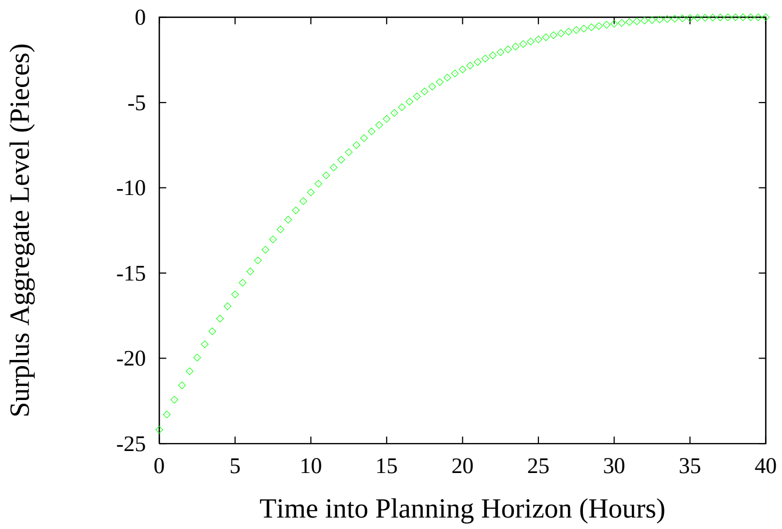


Figure 12: Surplus Aggregate Level for stage 3 with [3,5,4] active workstations.

Part 4. Conclusions

- The LQGP problem is a theoretical model and a canonical computational model having several exact analytical results, except for solving the LQGP nonlinear equation, and the auxiliary coefficient equations.

- The problem will be useful for severe testing of physical devices in computer experiments, investigating large fluctuations in financial markets, as well as many other applications, because the Poisson random component is useful for modeling large random fluctuations, whereas Gaussian noise is useful for relatively milder background fluctuations.

- The stochastic differential equation formulation used here makes the construction of models analogous to that for other dynamic systems.

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