Multistage Manufacturing System: Example of LQGP Problem *

J. J. Westman and F. B. Hanson
Laboratory for Advanced Computing
University of Illinois at Chicago
851 South Morgan St.; Room 322 SEO; M/C 249
Chicago, IL 60607-7045
e-mail: jwestman@math.uic.edu
hanson@math.uic.edu
web page: http://www.math.uic.edu/~hanson/MMS_Example.html
SC 1996

Abstract. The Linear Quadratic Gaussian Poisson (LQGP) problem denotes an optimal control problem with linear dynamics and quadratic costs with both Gaussian and Poisson noise disturbances (see Westman and Hanson [14]). A numerical example for a multistage manufacturing system (MMS) application of the LQGP problem is presented for the regular or unconstrained control problem. The MMS is a *hybrid system*, a combination of discrete (e.g., Markov chains or Poisson Noise) and continuous systems in an uncertain environment.

*Work supported in part by the National Science Foundation Grants DMS-9301107 and DMS-9626692. Computational support was provided by the National Scalable Cluster Project at the University of Illinois at Chicago.

Model Assumptions for MMS

I. The loading (input of raw materials) and unloading (delivery of finished goods) stages are not considered part of the MMS.

II. The MMS consists of k stages.

III. Stage j is preceded by stage $j - 1$ and followed by stage $j + 1$.

IV. For each stage, the machines are assumed to be identical in all properties and produce at the same rate.

V. The surplus aggregate level for stage ℓ, $a_\ell(t)$, is the number of pieces that have successfully completed ℓ stages of the manufacturing process that are not defective and not consumed by stage $\ell + 1$.

VI. For every stage, one or more machines may be repaired or fail at any given time depending on the number of operational machines at that time.

VII. The production level is expressed as the utilization (fraction of time busy) for each machine on a given stage.

VIII. Only the regular or unconstrained control is considered here, this may cause the production levels to exceed 1. In this case the manager should set the production level to 1 and adjust the surplus aggregate level accordingly.

Defining Stochastic Differential Equation

$$\begin{bmatrix} d\mathbf{a}(t) \\ d\mathbf{n}(t) \end{bmatrix} = \left[\begin{bmatrix} 0_{k\times k} & \mathrm{diag}[\mathbf{P}]\mathrm{diag}[\mathbf{c}(t)] \\ 0_{k\times k} & 0_{k\times k} \end{bmatrix} \begin{bmatrix} \mathbf{a}(t) \\ \mathbf{n}(t) \end{bmatrix} + \begin{bmatrix} I_{k\times k} \\ 0_{k\times k} \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} -\mathbf{d}(t) \\ \mathbf{0}_{k\times 1} \end{bmatrix} \right] dt$$
$$+ \begin{bmatrix} \mathrm{diag}[\mathbf{g}(t)] \\ 0_{k\times k} \end{bmatrix} d\mathbf{W}(t) + \begin{bmatrix} 0_{k\times k} & 0_{k\times k} \\ I_{k\times k} & -I_{k\times k} \end{bmatrix} \begin{bmatrix} d\mathbf{\Pi}^R(t) \\ d\mathbf{\Pi}^F(t) \end{bmatrix}$$

where $\mathrm{diag}[\mathbf{c}(t)] = [\Sigma_j \delta_{i,j} c_j]_{k\times k}$ is the diagonalization of the vector whose components are given by $c_i(t)$ for $i = 1$ to k

- $\mathbf{a}(t)$ is a $k \times 1$ vector for the state variable of the surplus aggregate level – the surplus or shortfall of production.
- $\mathbf{n}(t)$ is a $k \times 1$ vector for the state variable for the number of operational machines.
- $\mathbf{P}$ is a $k \times 1$ vector for the maximum production capacity.
- $\mathbf{c}(t)$ is a $k \times 1$ vector for the production level – $c_j(t)$ is the utilization of each machine for stage j at time t.
- $\mathbf{u}(t)$ is a $k \times 1$ vector for the control.
- $\mathbf{d}(t)$ is a $k \times 1$ vector for the demand rate.
- $\mathbf{g}(t)$ is a $k \times 1$ vector for the coefficient of the Gaussian Noise.
- $d\mathbf{W}(\mathbf{t})$ is a $k \times 1$ vector of Guassian noise.
- $d\mathbf{\Pi}^R(t)$ is a $k \times 1$ vector of Poisson noise for the repair process for the broken machines.
- $d\mathbf{\Pi}^F(t)$ is a $k \times 1$ vector of Poisson noise for the failure process for the operational machines.

Performance Index for MMS

$$V(\mathbf{X}(t), \mathbf{u}(t), t) = \frac{1}{2}\mathbf{X}^T(t_f)S(t_f)\mathbf{X}(t_f) + \frac{1}{2}\int_t^{t_f} \mathbf{u}^T(\tau)R(\tau)\mathbf{u}(\tau)d\tau$$

where

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{a}(t) \\ \\ \mathbf{n}(t) \end{bmatrix}$$

- The performance index to be minimized is a quadratic cost-to-go form.
- The salvage or terminal cost matrix S is a symmetric, positive semi-definite$2k \times 2k$ matrix.
- The salvage cost is based on the idea of *Just In Time* or stockless production (see Hall [7]). This means that a penalty is imposed if we have a surplus or shortfall of production at the end of the planning horizon in any stage of the manufacturing system.
- The instantaneous quadratic cost coefficient $R(t)$ is a symmetric, positive definite $k \times k$ matrix, so that the integral term in the performance index is quadratic in the control effort.

Numerical Example and Results

- The following graphs are for a MMS with 3 stages with a planning horizon of 40 hours and a production goal of 10,000 pieces. These graphs are for the second stage with 3, 4, and 5 operational machines respectively.
- The surplus aggregate level is assumed to be zero for the entire planning horizon.
- The production level is the utilization of all operational machines for a given stage.
- The virtual static production level is the rate of production not influenced by repairs or failures in the reamining time of the planning horizon.
- The production level adjusted with regular control anticipates repairs and failures in the remaining time of the planning horizon.
- The time is measured from the beginning of the planning horizon.
- The state ID describes the number of operational machines for all stages. See the "State ID Table".
- If the production level is greater than 1, a physically unrealizable situation, then the production level should be set to 1 and the surplus aggregate level should be adjusted to reflect the shortfall of production.

State ID Table for Stage 2

State ID	Number of Operational Machines		
	Stage 2	Stage 3	Stage 1
25	3	1	1
26	3	1	2
27	3	1	3
28	3	2	1
29	3	2	2
30	3	2	3
31	3	3	1
32	3	3	2
33	3	3	3
34	3	4	1
35	3	4	2
36	3	4	3
37	4	1	1
38	4	1	2
39	4	1	3
40	4	2	1
41	4	2	2
42	4	2	3
43	4	3	1
44	4	3	2
45	4	3	3
46	4	4	1
47	4	4	2
48	4	4	3
49	5	1	1
50	5	1	2
51	5	1	3
52	5	2	1
53	5	2	2
54	5	2	3
55	5	3	1
56	5	3	2
57	5	3	3
58	5	4	1
59	5	4	2
60	5	4	3

User Guide for Production Level Graphs

I. Determine the time, T, into the planning horizon.

II. Determine the number of operational machines for all stages at time T.

III. For each manufacturing stage, from the State ID Table for a given stage obtain the STATE ID.

IV. For each manufacturing stage, from the appropriate Production Level Graph for a given stage and number of operational machines, set the PRODUCTION LEVEL to the value for the Production Level Adjusted with Regular Control for time T and STATE ID.

V. For each manufacturing stage, if the PRODUCTION LEVEL is greater than 1 for a given stage, then set the PRODUCTION LEVEL to 1 and adjust the SURPLUS AGGREGATE LEVEL accordingly.

References

[1] R. Akella and P. R. Kumar, "Optimal control of production rate in a failure prone manufacturing system," *IEEE Trans. Autom. Control*, vol. 31, pp. 116-126, Feb. 1986.

[2] T. Bielecki and P. R. Kumar, "Optimality of zero-inventory policies for unreliable manufacturing systems," *Operations Research*, vol. 36, pp. 532-541, July-Aug. 1988.

[3] E. K. Boukas and A. Haurie, "Manufacturing flow control and preventive maintenance: A stochastic control approach," *IEEE Trans. Autom. Control*, vol. 35, pp. 1024-1031, Sept. 1990.

[4] E. K. Boukas and H. Yang, "Optimal control of manufacturing flow and preventive maintenance," *IEEE Trans. Autom. Control*, vol. 41, pp. 881-885, June 1996.

[5] A. E. Bryson and Y. Ho, *Applied Optimal Control*, Ginn, Waltham, 1969.

[6] C. Dupont-Gatelmand, "A survey of flexible manufacturing systems," *J. Manufacturing Systems*, vol. 1, 1, pp. 1-16, 1982.

[7] R. W. Hall, *Zero Inventories*, Dow Jones-Irwin, Homewood, Illinois, 1983.

[8] F. B. Hanson and D. Ryan, "Mean and quasi-deterministic equivalence for linear stochastic dynamics," *Math. Biosci.*, vol. 93, pp. 1-14, 1989.

[9] J. Kimemia and S. B. Gershwin, "An algorithm for the computer control of a flexible manufacturing system," *IIE Trans.*, vol. 15, pp. 353-362, Dec. 1983

[10] F. L. Lewis, *Optimal Estimation with an Introduction to Stochastic Control Theory*, Wiley, New York, 1986.

[11] M. Mariton, *Jump Linear Systems in Automatic Control*, Marcel Dekker, New York, 1990.

[12] K. Naimipour and F. B. Hanson, "Numerical convergence for the Bellman equation of stochastic optimal control with quadratic costs and constraints," *Dynamics and Control 3*, pp. 237-259, 1993.

[13] H. M. Taylor and S. Karlin, *An Introduction to Stochastic Modeling*, Academic Press, San Diego, 1984.

[14] J. J. Westman and F. B. Hanson, *The LQGP Problem: A Manufacturing Application*, submitted to the *Proc. of 1997 American Control Conference, Albuquerque, NM*, 12 pages in ms.

[15] W. M. Wonham, "Random differential equations in control theory," in *Probabilistic Methods in Applied Mathematics*, vol. 2, edited by A. T. Bharucha-Reid, Academic Press, New York, pp. 131-212, 1970.