FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2010

Lecture 1 Homework: Introduction with Sampling

(due by Lecture 2 in Chalk FINM331 Assignments submenu)

- **You must show your work, code and/or worksheet for full credit.**
- **Justifying each non-trivial step with a reason is part of showing your work.**
- **There are 10 points per question if correct and best answer.**
- **There are negative points for missing homework sets.**

1. If X and Y are independent random variables with means (μ_X, μ_Y) and variances (σ_X^2, σ_Y^2), respectively, and $Z = X - Y$ find $\text{Cov}[X, Z]$, the correlation between X and Z, and the correlation between Y and Z. (*Adapted from Rice, p. 170, problem 47.*)

2. Use the completing the square technique to evaluate the expectation $\text{E}_X[\exp(\alpha + \beta X)]$ with respect to the normal density $f_X^{(n)}(x; \mu, \sigma^2)$, i.e., with mean μ and variance σ^2, for constants (α,β). (*See Lecture 1, pp. 39ff.*)

3. Suppose that a measurement has mean $\mu = 0.035$ and $\sigma = 0.25$. Let $\overline{X}$ be the average of n such measurements. Compute estimates, using the CLT, how large should n be so that $\text{Prob}(|\overline{X} - \mu| < 2\sigma] \geq \text{pci}$ for each of the four (4) values $\text{pci} = [0.95, 0.975, 0.99, 0.995]$. (*Adapted from Rice, p. 189, problem 17, but see pp. 184ff and Lecture 1, p. 46.*)

4. Select a standardized IID RVs X_i for $i = 1 : n$ that can be used to form a proper sample mean for the Central Limit Theorem (CLT), show that the X_i are independent, and find the standardized IID RVs appropriate for the CLT in each case. (See also, Rice, pp. 188ff.)
Do this for

(a) The additive asset model of *Lecture 1, Sect. 1.4, p. 23, eq. (19).*

(b) The multiplicative asset model of *Lecture 1, Sect. 1.5, p. 27, eq. (25).*

5. Compute the Monte Carlo estimate for the risk-neutral pricing of aEuropean put option, i.e., with discounted payoff $\exp(-rT)\max(K - A, 0)$ where K is the strike price, T is the expiration date, A is the asset price underlying the option, and r is the risk-free interest rate; use the parameter values $\{A0 = 110; K = 100; r = 0.035; std = 0.25; T = 0.5; N = 2.e5;\}$ for four (4) confidence interval percentiles $PC = [0.95, 0.975, 0.99, 0.995]$. (See Lecture 1, p. 66, code `mcm4eurocall.m`.)