Lecture 2 Homework:

(due by Lecture 3 in Chalk FINM331 Assignments submenu)

• You must show your work, code and/or worksheet for full credit.
• Justifying each non-trivial step with a reason is part of showing your work.
• There are 10-20 points per question if correct and best answer.
• Report numerical values in at least 4 significant digits (e.g., for errors use format like \( \%8.3e \)).

1. Compute the Monte Carlo estimate for the risk-neutral pricing of a European put option, i.e., with discounted payoff \( \exp(-rT) \max(K-A,0) \) where \( K \) is the strike price, \( T \) is the expiration date, \( A \) is the asset price underlying the option, and \( r \) is the risk-free interest rate; use the parameter values \{\( A_0 = 110; K = 100; r = 0.035; std = 0.25; T = 0.5; N = 2.e5; \} \) for four (4) confidence interval percentiles \( PC = [0.95, 0.975, 0.99, 0.995] \). (See Lecture 2, p. 18 and Chalk/CourseDocs, code \texttt{mcm4eurocall.m}) \((10\text{ points})\)

2. Compute the normalization constant \( C^{(ac)} \), the mean \( \mu^{(ac)} \) and standard deviation \( \sigma^{(ac)} \) by the normal Monte Carlo method a illustrated in Lecture 2, pp. 14-16 accept-reject code (abbreviated version) \texttt{mcm2acceptreject09.m}, with complete test version in Chalk/CourseDocs. In particular, the acceptable or truncated density \( f_X^{(ac)}(x) = f_X(x)/C^{(ac)} \) on \( (c,d) \), where \( f_X(x) \) is a normal probability density with mean \( \mu \) and standard deviation \( \sigma \). Let \( \mu = 2.4e-4 \), \( \sigma = 0.011 \), \( c = -0.095 \) and \( d = 0.110 \). \((20\text{ points})\)

   (a) Show that \( C^{(ac)} = \int_c^d f_X(x)dx \) and use MATLAB’s \texttt{normcdf} to compute a benchmark numerical value.

   (b) Modify the cited code to the extent necessary. Use sample sizes of \( n = \{10^2, 10^3, 10^4, 10^5\} \) for the estimates expectations of each of \( g(x) = \{1, x, (x-\mu^{(ac)})^2\} \) on the accepted interval \( (c,d) \), i.e, for the \( g(x)1^{(ac)}(x) \), where \( 1^{(ac)}(x) \) is the indicator function for \( x \in (c,d) \), to obtain Monte Carlo estimates of \( C^{(ac)}, \mu^{(ac)} \) and \( (\sigma^{(ac)})^2 \), respectively. Use the estimate of \( (\sigma^{(ac)})^2 \) to compute the estimate of the standard error for the estimate of \( \mu^{(ac)} \) (Lect. 2, p. 3, 15-17).

   (c) Compare relative differences between the estimates of \( C^{(ac)} \) from parts (b) relative to (a), means \( \mu^{(ac)} \) relative to \( \mu \) and standard deviations \( \sigma^{(ac)} \) relative to \( \sigma \), for each \( n \). Tabulate, plot and discuss the estimated accepted results.

3. Consider the small time step, \( \Delta t \ll 1 \), so the zero-one Bernoulli jump law is valid for the jump part of the log-return asset time-series jump-diffusion model from Lecture 2, Eq. (28),

\[
LR_i = \log(A_{i+1}) - \log(A_i) \stackrel{\Delta t}{\equiv} \sigma \sqrt{\Delta t} Z_i + (\mu - \sigma^2 Z_i^2/2) \Delta t + Q_i Y_i,
\]

where \( Z_i \) are IID normal with zero-mean and unit-variance, \( Y_i \) are IID Poisson with \( \lambda \Delta t \) mean and variance, and \( Q_i \equiv \log(1 + \nu_i) \) are IID with \( \mu_Q \)-mean and \( \sigma_Q \)-variance. All three IID RVs are pairwise independent. The parameters \{\( \mu, \sigma, \lambda, \mu_Q, \sigma_Q \)\} are constants. Find \( \mathbb{E}[LR_i] \) and \( \text{Var}[LR_i] \) to \( O(\Delta t) \), i.e., neglect any power of \( \Delta t \) greater than one. \{Hint: \( \text{Var}[LR_i] \) is easier if expanded in deviations from the mean.\} \((10\text{ points})\)