FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2010

Lecture 2 Homework:

(due by Lecture 3 in Chalk FINM331 Assignments submenu)

- **You must show your work, code and/or worksheet for full credit.**
- **Justifying each non-trivial step with a reason is part of showing your work.**
- **There are 10-20 points per question if correct and best answer.**
- **Report numerical values in at least 4 significant digits (e.g., for errors use format like `%8.3e`).**

1. Compute the Monte Carlo estimate for the risk-neutral pricing of a European put option, i.e., with discounted payoff $\exp(-rT)\max(K-A,0)$ where K is the strike price, T is the expiration date, A is the asset price underlying the option, and r is the risk-free interest rate; use the parameter values $\{A0 = 110; K = 100; r = 0.035; std = 0.25; T = 0.5; N = 2.e5;\}$ for four (4) confidence interval percentiles $PC = [0.95, 0.975, 0.99, 0.995]$. (See Lecture 2, p. 18 and Chalk/CourseDocs, code `mcm4eurocall.m`.) *(10 points)*

2. Compute the normalization constant $C^{(\mathrm{ac})}$, the mean $\mu^{(\mathrm{ac})}$ and standard deviation $\sigma^{(\mathrm{ac})}$ by the normal Monte Carlo method a illustrated in Lecture 2, pp. 14-16 accept-reject code (abbreviated version) `mcm2acceptreject09.m`, with complete test version in Chalk/CourseDocs. In particular, the acceptable or truncated density $f_X^{(\mathrm{ac})}(x) = f_X(x)/C^{(\mathrm{ac})}$ on (c,d), where $f_X(x)$ is a normal probability density with mean μ and standard deviation σ. Let $\mu = 2.4\text{e-}4$, $\sigma = 0.011$, $c = -0.095$ and $d = 0.110$. *(20 points)*

(a) Show that $C^{(\mathrm{ac})} = \int_c^d f_X(x)dx$ and use MATLAB's `normcdf` to compute a benchmark numerical value.

(b) Modify the cited code to the extent necessary. Use sample sizes of $n = \{10^2, 10^3, 10^4, 10^5\}$ for the estimates expectations of each of $g(x) = \{1, x, (x-\mu^{(\mathrm{ac})})^2\}$ on the accepted interval (c,d), i.e, for the $g(x)\mathbf{1}^{(\mathrm{ac})}(x)$, where $\mathbf{1}^{(\mathrm{ac})}(x)$ is the indicator function for $x \in (c,d)$, to obtain Monte Carlo estimates of $C^{(\mathrm{ac})}$, $\mu^{(\mathrm{ac})}$ and $(\sigma^{(\mathrm{ac})})^2$, respectively. Use the estimate of $(\sigma^{(\mathrm{ac})})^2$ to compute the estimate of the standard error for the estimate of $\mu^{(\mathrm{ac})}$ (Lect. 2, p. 3, 15-17).

(c) Compare relative differences between the estimates of $C^{(\mathrm{ac})}$ from parts (b) relative to (a), means $\mu^{(\mathrm{ac})}$ relative to μ and standard deviations $\sigma^{(\mathrm{ac})}$ relative to σ, for each n. Tabulate, plot and discuss the estimated accepted results.

3. Consider the small time step, $\Delta t \ll 1$, so the zero-one Bernoulli jump law is valid for the jump part of the log-return asset time-series jump-diffusion model from Lecture 2, Eq. (28),

$$\mathrm{LR}_i = \log(A_{i+1}) - \log(A_i) \overset{\Delta t}{=} \sigma\sqrt{\Delta t}Z_i + (\mu - \sigma^2 Z_i^2/2)\Delta t + Q_i Y_i,$$

where Z_i are IID normal with zero-mean and unit-variance, Y_i are IID Poisson with $\lambda\Delta t$ mean and variance, and $Q_i \equiv \log(1+\nu_i)$ are IID with μ_Q-mean and σ_Q-variance. All three IID RVs are pairwise independent. The parameters $\{\mu, \sigma, \lambda, \mu_Q, \sigma_Q\}$ are constants. Find $\mathrm{E}[\mathrm{LR}_i]$ and $\mathrm{Var}[\mathrm{LR}_i]$ to $\mathrm{O}(\Delta t)$, i.e., neglect any power of Δt greater than one. $\{$*Hint:* $\mathrm{Var}[\mathrm{LR}_i]$ *is easier if expanded in deviations from the mean.*$\}$ *(10 points)*