FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2010
Lecture 4 Homework:
(due by Lecture 5 in Chalk FINM331 Assignments submenu)

- **You must show your work, code and/or worksheet for full credit.**
- **Justifying each non-trivial step with a reason is part of showing your work.**
- **There are 10 or more points per question if correct and best answer.**
- **Report numerical values in at least 4 significant digits (e.g., for errors use format like `%8.3e`).**

1. (10 points) Let $X_0 \stackrel{\text{dist}}{=} \mathcal{N}(0,1)$, i.e., a standard normal, and $\text{Prob}[Z_i = -1] = 0.5 = \text{Prob}[Z_i = +1]$ for $i = 1:2$ be three independent RVs. Then, define $X_1 \equiv Z_1 \cdot X_0$ and $X_2 \equiv Z_2 \cdot X_0$.

(a) Prove that $X_1 \stackrel{\text{dist}}{=} \mathcal{N}(0,1)$, $X_2 \stackrel{\text{dist}}{=} \mathcal{N}(0,1)$ and that $\rho_{X_1,X_2} = 0$.

(b) Show that X_1 and X_2 are **NOT independent**.

Remark: This problem is is a standard counterexample problem that zero correlation does not imply independence, in general. (Adapted from Carmona (2004) p.100)

2. (20 points) In order to convert an invertible, fully infinite domain distribution $F_X(x;\vec{\theta})$, with parameter vector $\vec{\theta}$, to a realistic, renormalized finite domain (FD) distribution, on $[R_1, R_2]$, $R_1 < \mu < R_2$, with basic statistics $(\mu, \sigma^2) \in \vec{\theta}$,

$$u = F_X^{(\text{fd})}(x;\vec{\theta}, R_1, R_2) \equiv \left(F_X(x;\vec{\theta}) - F_X(R_1;\vec{\theta})\right) \Big/ \left(F_X(R_2;\vec{\theta}) - F_X(R_1;\vec{\theta})\right), \quad (1)$$

where u is clearly a standard uniform sample variable, i.e., on $(0,1)$.

(a) Show that the finite domain sample variable x on (R_1, R_2) is related to the standard uniform sample variable by

$$x = (F_X)^{-1}\left(\left(F_X\left(R_2;\vec{\theta}\right) - F_X\left(R_1;\vec{\theta}\right)\right) \cdot u + F_X\left(R_1;\vec{\theta}\right)\right). \quad (2)$$

(b) In general, the properties of the parameter vector will not be preserved except partially in the some cases with symmetric ranges, so show this for the case of the finite-domain normal (FDN) with statistics $(\mu^{(\text{fdn})}, (\sigma^{(\text{fdn})})^2)$ by showing

$$\mu^{(\text{fdn})} = \mu - \sigma^2 (f_2 - f_1)/F_{12} \quad (3)$$

where $F_1 \equiv F_X^{(\text{n})}(R_1;\mu,\sigma^2)$, $F_2 \equiv F_X^{(\text{n})}(R_2;\mu,\sigma^2)$, $F_{12} \equiv F_2 - F_1$, $f_1 \equiv f_X^{(\text{n})}(R_1;\mu,\sigma^2)$, $f_2 \equiv f_X^{(\text{n})}(R_2;\mu,\sigma^2)$, and

$$\begin{aligned}\left(\sigma^{(\text{fdn})}\right)^2 = (\mu - \mu^{(\text{fdn})})^2 + \sigma^2 \big(1 - ((R_2 - \mu) f_2 + (\mu - R_1) f_1)/F_{12} \\ -2(\mu - \mu^{(\text{fdn})})(f_2 - f_1)/F_{12}\big)\end{aligned} \quad (4)$$

{*Hint: Try integration by parts only with powers of* $(x - \mu)$ *multiplying the density.*}

(c) Show, if R_1 and R_2 are symmetrically located about the mean μ, then $\mu^{(\text{fdn})} = \mu$, but that

$$\left(\sigma^{(\text{fdn})}\right)^2 = \sigma^2 (1 - 2(R_2 - \mu) f_2 / F_{12}) \le \sigma^2. \quad (5)$$

3. (20 points) For the 2009 S&P 500 Index data log-returns of Homework 3 – Problem 1, pick the **POT** (peak over threshold value) with sufficient tail count for the left tail, so that a GP fit function is viable. Then (1) extract the values less than or equal to the POT into a vector, (2) reverse the sign, and (3) sort the values in ascending order. Then,

(a) Display and hold on to this sorted tail vector with a histogram.

(b) Use the GP distribution function `gpfit.m`, or equivalent, to fit to a GP power law. Display the fit GP density scaled to plot with the histogram of part (a) on its scale for comparison.

(c) use the fast exponential analysis function `expan.m` directly or the class modification or equivalent to fit to an exponential. Display separately.

Discuss the results.