FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2010

Lecture 4 Homework:

1. (10 points) Let $X_0 \sim \mathcal{N}(0, 1)$, i.e., a standard normal, and $\text{Prob}[Z_i = -1] = 0.5 = \text{Prob}[Z_i = +1]$ for $i = 1:2$ be three independent RVs. Then, define $X_1 \equiv Z_1 \cdot X_0$ and $X_2 \equiv Z_2 \cdot X_0$.

   (a) Prove that $X_1 \sim \mathcal{N}(0, 1)$, $X_2 \sim \mathcal{N}(0, 1)$ and that $\rho_{X_1, X_2} = 0$.

   (b) Show that $X_1$ and $X_2$ are NOT independent.

Remark: This problem is is a standard counterexample problem that zero correlation does not imply independence, in general. (Adapted from Carmona (2004) p.100)

2. (20 points) In order to convert an invertible, fully infinite domain distribution $F_X(x; \vec{\theta})$, with parameter vector $\vec{\theta}$, to a realistic, renormalized finite domain (FD) distribution, on $[R_1, R_2]$, $R_1 < \mu < R_2$, with basic statistics $(\mu, \sigma^2) \in \vec{\theta}$,

   
   $$u = F_X^{(fd)}(x; \vec{\theta}, R_1, R_2) \equiv \left( F_X(x; \vec{\theta}) - F_X(R_1; \vec{\theta}) \right) \left/ \left( F_X(R_2; \vec{\theta}) - F_X(R_1; \vec{\theta}) \right) \right.,$$

   where $u$ is clearly a standard uniform sample variable, i.e., on $(0, 1)$.

   (a) Show that the finite domain sample variable $x$ on $(R_1, R_2)$ is related to the standard uniform sample variable by

   $$x = (F_X)^{-1}\left( F_X(R_2; \vec{\theta}) - F_X(R_1; \vec{\theta}) \right) \cdot u + F_X(R_1; \vec{\theta}).$$

   (b) In general, the properties of the parameter vector will not be preserved except partially in the some cases with symmetric ranges, so show this for the case of the finite-domain normal (FDN) with statistics $(\mu^{(fdn)}, (\sigma^{(fdn)})^2)$ by showing

   $$\mu^{(fdn)} = \mu - \sigma^2(f_2 - f_1)/F_{12}$$

   where $F_1 \equiv F_X^{(n)}(R_1; \mu, \sigma^2)$, $F_2 \equiv F_X^{(n)}(R_2; \mu, \sigma^2)$, $F_{12} \equiv F_2 - F_1$, $f_1 \equiv f_X^{(n)}(R_1; \mu, \sigma^2)$, $f_2 \equiv f_X^{(n)}(R_2; \mu, \sigma^2)$, and

   $$\left(\sigma^{(fdn)}\right)^2 = (\mu - \mu^{(fdn)})^2 + \sigma^2(1 - ((R_2 - \mu)f_2 + (\mu - R_1)f_1)/F_{12} - 2(\mu - \mu^{(fdn)})(f_2 - f_1)/F_{12})$$

   {Hint: Try integration by parts only with powers of $(x-\mu)$ multiplying the density.}

   (c) Show, if $R_1$ and $R_2$ are symmetrically located about the mean $\mu$, then $\mu^{(fdn)} = \mu$, but that

   $$\left(\sigma^{(fdn)}\right)^2 = \sigma^2 \left( 1 - 2(R_2 - \mu)f_2/F_{12} \right) \leq \sigma^2.$$  


3. (20 points) For the 2009 S&P 500 Index data log-returns of Homework 3 – Problem 1, pick the POT (peak over threshold value) with sufficient tail count for the left tail, so that a GP fit function is viable. Then (1) extract the values less than or equal to the POT into a vector, (2) reverse the sign, and (3) sort the values in ascending order. Then,

(a) Display and hold on to this sorted tail vector with a histogram.

(b) Use the GP distribution function `gpfit.m`, or equivalent, to fit to a GP power law. Display the fit GP density scaled to plot with the histogram of part (a) on its scale for comparison.

(c) use the fast exponential analysis function `expan.m` directly or the class modification or equivalent to fit to an exponential. Display separately. Discuss the results.