1. (20 points) Bayesian Normal Likelihood with Proper Normal-Gamma Prior

Test the Bayesian estimate on pp. 50-51 of Lecture 8. Assume reasonable guestimates for the prior parameters $\mu_0, \gamma, \lambda, \alpha$. Use the data of the 2009 S&P 500 Index log-returns for the data likelihood specification, which also might help with the guestimates.

(a) Using the posterior density, find the maximum likelihood or mode estimates for each of the two normal likelihood parameters $(\mu, \xi = 1/\sigma^2)$ analytically in terms of critical point formulas. Then find numerical evaluations of the estimates $(\hat{\mu}, \hat{\xi} = 1/\hat{\sigma}^2)$.

(b) Plot the marginal distributions for the desired parameters $(\mu, \xi)$. Also, give the standard errors for each.

*Hint:* For $\mu$-marginal, the $\xi$ is distributed as a Gamma variate and for the $\xi$-marginal, the $\mu$ is distributed as a Normal variate.