

Lecture 4 Homework: Stochastic Jump and Jump-Diffusion Processes

(Due by Lecture 5 in Chalk FINM345 Assignment Submenu)

{Note dropping the Digital Dropbox}

You must show your work, code and/or worksheet for full credit.

There are 10 points per question if correct answer.

1. Inverse Problem for Poisson Integral.

Find $X(t) = F(P(t))$ if

$$\int_0^t X(s)dP(s) = e^{cP(t)} \ln(aP(t) + b) - \ln(b) , \quad (1)$$

where a , b and c are real constants.

2. Solution to a Jump SDE.

Solve the following jump SDE for $X(t)$, $E[X(t)]$, and $\text{Var}[X(t)]$,

$$dX(t) = aX^2(t)dt - \frac{bX^2(t)}{1 + bX(t)}dP(t) , \quad (2)$$

where $a < 0$, $b > 0$ and $c > 0$ are constants such that $E[P(t)] = ct$, while $X(0) = x_0 > 0$, with probability one.

{Hint: seek a transformation $Y(t) = F(X(t))$ for some F such that $Y(t)$ satisfies a constant coefficient SDE. The answer may be left as a Poisson distribution sum.}

3. Solution to another Jump SDE.. Solve the following Poisson jump SDE for $X(t)$ and $E[X(t)]$:

$$dX(t) = a\sqrt{X(t)}dt + b \left(b + 2\sqrt{X(t)} \right) dP(t),$$

where $E[P(t)] = \lambda_0 t$ and $X(0) = x_0 > 0$, with probability one, while λ_0 , a and b are real constants.

(Hint: Find a power transformation to convert the SDE to a constant coefficient SDE.)

4. For Jump-Diffusion SDE, Find Coefficients Transformable to Constant Coefficient SDE. Show that the (Itô) jump-diffusion SDE for $X(t)$,

$$dX(t) = f(X(t))dt + bX^a(t)dW(t) + h(X(t))dP(t) , \quad (3)$$

can be transformed by $Y(t) = F(X(t))$ to a **constant coefficient SDE**, where b and $a \neq 1$ are real constants, and $X(0) = x_0 > 0$, with probability one. In a proper answer, derive the power forms of $f(X(t))$ and $h(X(t))$ from the constant coefficient SDE conditions. Also, what is the answer when $a = 1$?