1. Inverse Problem for Poisson Integral.

Find \( X(t) = F(P(t)) \) if
\[
\int_{0}^{t} X(s) dP(s) = e^{cP(t)} \ln(aP(t) + b) - \ln(b),
\]
where \( a, b \) and \( c \) are real constants.

2. Solution to a Jump SDE.

Solve the following jump SDE for \( X(t), \ E[X(t)], \) and \( \text{Var}[X(t)], \)
\[
dX(t) = aX^2(t)dt - \frac{bX(t)}{1+bX(t)} dP(t),
\]
where \( a < 0, \ b > 0 \) and \( c > 0 \) are constants such that \( E[P(t)] = ct \), while \( X(0) = x_0 > 0 \), with probability one.

\{Hint: seek a transformation \( Y(t) = F(X(t)) \) for some \( F \) such that \( Y(t) \) satisfies a constant coefficient SDE. The answer may be left as a Poisson distribution sum.\}

3. Solution to another Jump SDE.

Solve the following Poisson jump SDE for \( X(t) \) and \( E[X(t)] \):
\[
dX(t) = a\sqrt{X(t)}dt + b \left( b + 2\sqrt{X(t)} \right) dP(t),
\]
where \( E[P(t)] = \lambda_0 t \) and \( X(0) = x_0 > 0 \), with probability one, while \( \lambda_0, \ a \) and \( b \) are real constants.

\{Hint: Find a power transformation to convert the SDE to a constant coefficient SDE.\}

4. For Jump-Diffusion SDE, Find Coefficients Transformable to Constant Coefficient SDE.

Show that the (Itô) jump-diffusion SDE for \( X(t), \)
\[
dX(t) = f(X(t))dt + bX^a(t)dW(t) + h(X(t))dP(t),
\]
can be transformed by \( Y(t) = F(X(t)) \) to a constant coefficient SDE, where \( b \) and \( a \neq 1 \) are real constants, and \( X(0) = x_0 > 0 \), with probability one. In a proper answer, derive the power forms of \( f(X(t)) \) and \( h(X(t)) \) from the constant coefficient SDE conditions. Also, what is the answer when \( a = 1? \)