FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009

Lecture 5 Homework BackUpOnly (HW5BU) : Simple and Compound Jump-Diffusions Stochastic Calculus

(Due by Lecture 6 in Chalk FINM345 Assignment Submenu)

{Note: Dropped the Digital Dropbox}

You must show your work, code and/or worksheet for full credit. There are 10 points per question if correct answer and negative points for missing homework sets.

1. **Basic Statistics of Linear Simple Jump-Diffusion SDEs with Variable Coefficients:**

   - Complete the algebraic IFA exercise for the expectation exercise Theorem 5.1.3, p. L5-p25 (Th. 4.31, p. 119, textbook).
   - Formally prove the following IFA variance theorem using Lemmas 5.1.1 on p. L5-p21 and 5.1.2 on p. L5-p23:
     Let $\mu(t)$, $\sigma^2(t)$ and $\lambda(t)\nu^j(t)$ for $j = 1:2$ be integrable on $[t_0, t]$. Then

$$\begin{aligned}\mathrm{Var}[X(t)] &\overset{\text{ifa}}{=} x_0^2 \exp\left(2\int_{t_0}^{t}(\mu(s)+\lambda(s)\nu(s))\,ds\right)\\ &\cdot\left(\exp\left(\int_{t_0}^{t}\left(\sigma^2(s)+\lambda(s)\nu^2(s)\right)ds\right)-1\right)\end{aligned} \tag{1}$$

     for the state trajectory $X(t)$ given in Eq. **(4.37)** on p. L4-p50 ((4.78) p. 110 textbook).

     {*Hint: first show partial results by separately showing the the expectation results for the squared diffusion and jump exponents parts as for the original exponentials in diffusion exponenial Lemma 5.1.1 and jump exponential lemma 5.1.2, then putting the results together by the well-know variance-moments identity.*}

2. Simulate $X(t)$ for the linear simple jump-diffusion with constant coefficients mean $\mu_0 = 0.285$, variance $\sigma_0^2 = 2.25$ $\lambda_0 = 3.45$ per year, $\nu_0 = -0.215$, $X(0) = 0.5$, $t_0 = 0$, final time $T = 2.0$ years, $N = 2,000$ time-steps per state for M = 4 diffusion and sequential bino jump states.

   {*Hint: Modify the linear jump-diffusion SDE simulator* ***linjumpdiff09fig1.m*** *on pp. L5-p11ff and found on Chalk/CourseDocuments.*}

3. For the log-double-uniform jump distribution,

$$\phi_Q(q;t) \equiv \left\{ \begin{array}{ll} 0, & -\infty < q < a(t) \\ p_1(t)/|a|(t), & a(t) \le q < 0 \\ p_2(t)/b(t), & 0 \le q \le b(t) \\ 0, & b(t) < q < +\infty \end{array} \right\}, \tag{2}$$

where $p_1(t)$ is the probability of a negative jump and $p_2(t)$ is the probability of a positive jump on $a(t) < 0 \le b(t)$, show that

(a) $\mathrm{E}_Q[Q] = \mu_j(t) = (p_1(t)a(t) + p_2(t)b(t))/2$;

(b) $\mathrm{Var}_Q[Q] = \sigma_j^2(t) = (p_1(t)a^2(t) + p_2(t)b^2(t))/3 - \mu_j^2(t)$;

(c) $E_Q[(Q - \mu_j(t))^3] = (p_1(t)a^3(t) + p_2(t)b^3(t))/4 - \mu_j(t)(3\sigma_j^2(t) + \mu_j^2(t))$;

(d) $\mathrm{E}[\nu(Q)] = \mathrm{E}[\exp(Q) - 1]$, where the answer needs to be derived.

4. General exponential-expectation interchange formula for linear jump-diffusions. Formally show that

$$\mathrm{E}\left[\exp\left(\int_0^t d\ln(X)(s)\right)\right] \overset{\text{ifa}}{=} \exp\left(\mathrm{E}\left[\int_0^t \left(\frac{dX}{X}\right)(s)\right]\right) \tag{3}$$

if $X(t)$ is a linear jump-diffusion process, **(4.34)** on p. L4-**p48** or (4.75; p. 109, textbook), verifying that both sides of (3) are equivalent. Assume all integrals of process coefficients are bounded and $X(t) > 0$.