

PSti1l

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Directions: Answer **All Questions** and show **All Work** in the **Exam Booklet** provided. Write your **Name**, **Social Security Number**, and **Discussion Section Hour/Day** on the Exam Book Cover Page. **Keep your eyes on your own work and keep your own work covered.** See *Table of Laplace Transforms* on Back.

1. (a) $\mathcal{L} [7e^{-8t} \cos(3t) + t^5 + 11t^2\delta(t - 3)]$
 (b) $\mathcal{L}^{-1} \left[\frac{e^{-2s}}{s^2 + 12s + 32} \right]$ (25 points total)
 (c) $\mathcal{L}^{-1} \left[\frac{s}{s^2 + 6s + 13} \right]$

2. (a) Find the general solution $y(x)$:

$$4x^2y''(x) + 8xy'(x) + y(x) = 0, \quad x > 0.$$

- (b) Find $z(t)$: (25 points total)

$$z''(t) + 3z'(t) + 2z(t) = 4\delta(t - \epsilon), \quad z(0) = 0, \quad z'(0) = 5, \quad \epsilon > 0.$$

What happens as $\epsilon \rightarrow 0^+$?

3. (a) Find a power series approximation to the solution $y(x)$ of the IVP:

$$y''(x) + 2xy'(x) - 4y(x) = 0, \quad y(0) = 2, \quad y'(0) = 1,$$

by assuming an expansion about $x = 0$ of the form $y(x) \simeq a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$, finding the coefficients $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. You do not need any more terms!

- (b) Classify all finite non-negative points of the ODE: (25 points total)

$$x^2y''(x) + xy'(x) + (x^2 - 1/9)y(x) = 0, \quad x > 0$$

and then give only the general form of the power series solution in the regular case (say about $x = a$) and the singular case, but do not evaluate any coefficients.

4. Solve the integral equation for $y(t)$: (25 points total)

$$y(t) + 8 \int_0^t y(v) \sin(t - v) dv = 9, \quad t \geq 0.$$

Table of Laplace Transforms¹

$f(t) = \mathcal{L}^{-1}(s)](t)$	$F(s) = \mathcal{L}[f(t)](s)$
$f(at)$	$\frac{1}{a}F\left(\frac{1}{a}\right)$
$e^{at}f(t)$	$F(s - a)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$\frac{1}{t}f(t), f(0) = 0$	$\int_s^\infty F(u)du$
$\int_0^t f(v)dv$	$F(s)/s$
$(f * g)(t)$	$F(s) \cdot G(s)$
$f(t - a)u(t - a), a \geq 0$	$e^{-as}F(s)$
$g(t)u(t - a), a \geq 0$	$e^{-as}\mathcal{L}[g(t + a)](s)$
$e^{at} \sin(bt)$	$\frac{b}{(s - a)^2 + b^2}$
$e^{at} \cos(bt)$	$\frac{s - a}{(s - a)^2 + b^2}$
$\sinh(bt)$	$\frac{b}{s^2 - b^2}$
$\cosh(bt)$	$\frac{s}{s^2 - b^2}$

¹From Back Cover of Nagle and Saff Text