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MATH 220 E1 EXAM 2 12pm 09 April 1999 Hanson

Laplace Transform and Power Series Solutions of ODEs

Directions: Answer **All Questions** and show **All Work** in the **Exam Booklet** provided. Write your **Name**, **Social Security Number**, and **Discussion Section Hour/Day** on the Exam Book Cover Page. **Keep your eyes on your own work and keep your own work covered.** See *Table of Laplace Transforms* on Back.

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1. (a) $\mathcal{L}\left[\mathbf{7e^{-8t}}\cos(\mathbf{3t})+\mathbf{t^5}+\mathbf{11t^2}\delta(\mathbf{t-3})\right]$

   (b) $\mathcal{L}^{\mathbf{-1}}\left[\dfrac{\mathbf{e^{-2s}}}{\mathbf{s^2+12s+32}}\right]$ (25 points total)

   (c) $\mathcal{L}^{\mathbf{-1}}\left[\dfrac{\mathbf{s}}{\mathbf{s^2+6s+13}}\right]$

2. (a) Find the general solution $y(x)$:

$$\mathbf{4x^2y''(x)+8xy'(x)+y(x)=0,\quad x>0.}$$

   (b) Find $z(t)$: (25 points total)

$$\mathbf{z''(t)+3z'(t)+2z(t)=4}\delta(\mathbf{t}-\epsilon),\quad \mathbf{z(0)=0},\quad \mathbf{z'(0)=5},\quad \epsilon>\mathbf{0}.$$

   What happens as $\epsilon \longrightarrow 0^+$?

3. (a) Find a power series approximation to the solution $y(x)$ of the IVP:

$$\mathbf{y''(x)+2xy'(x)-4y(x)=0},\quad \mathbf{y(0)=2},\quad \mathbf{y'(0)=1},$$

   by assuming an expansion about $x=0$ of the form $\mathbf{y(x)\simeq a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5}$, finding the coefficients $\{a_0,a_1,a_2,a_3,a_4,a_5\}$. You do not need any more terms!

   (b) Classify all finite non-negative points of the ODE: (25 points total)

$$\mathbf{x^2y''(x)+xy'(x)+(x^2-1/9)y(x)=0},\quad \mathbf{x>0}$$

   and then give only the general form of the power series solution in the regular case (say about $x=a$) and the singular case, but do not evaluate any coefficients.

4. Solve the integral equation for $y(t)$: (25 points total)

$$\mathbf{y(t)+8\int_0^t y(v)\sin(t-v)dv=9},\quad \mathbf{t\geq 0}.$$

Table of Laplace Transforms[1]

$f(t) = \mathcal{L}^{-1}(s)](t)$	$F(s) = \mathcal{L}[f(t)](s)$
$f(at)$	$\frac{1}{a}F\left(\frac{1}{a}\right)$
$e^{at}f(t)$	$F(s-a)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^nf(t)$	$(-1)^nF^{(n)}(s)$
$\frac{1}{t}f(t), f(0) = 0$	$\int_s^\infty F(u)du$
$\int_0^t f(v)dv$	$F(s)/s$
$(f * g)(t)$	$F(s) \cdot G(s)$
$f(t-a)u(t-a),\ a \geq 0$	$e^{-as}F(s)$
$g(t)u(t-a),\ a \geq 0$	$e^{-as}\mathcal{L}[g(t+a)](s)$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(bt)$	$\frac{b}{s^2-b^2}$
$\cosh(bt)$	$\frac{s}{s^2-b^2}$

[1] From Back Cover of Nagle and Saff Text