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MATH 220 E1 **EXAM 2** **12pm 09 April 1999** **Hanson**

Laplace Transform and Power Series Solutions of ODEs

Directions: Answer **All Questions** and show **All Work** in the **Exam Booklet** provided. Write your **Name**, **Social Security Number**, and **Discussion Section Hour/Day** on the Exam Book Cover Page. **Keep your eyes on your own work and keep your own work covered.** See *Table of Laplace Transforms* on Back.

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1. (a) $\mathcal{L}\left[\mathbf{7e^{-8t}\cos(2t) + t^5 + 6t^2\delta(t-4)}\right]$

   (b) $\mathcal{L}^{-1}\left[\dfrac{\mathbf{s}}{\mathbf{s^2+4s+29}}\right]$ (25 points total)

   (c) $\mathcal{L}^{-1}\left[\dfrac{\mathbf{e^{-3s}}}{\mathbf{s^2+9s+18}}\right]$

2. Solve the integral equation for $y(t)$: (25 points total)

$$\mathbf{y(t) + 3\int_0^t y(v)\sin(t-v)dv = 12, \quad t \geq 0.}$$

3. (a) Find $z(t)$: (25 points total)

$$\mathbf{z''(t) + 5z'(t) + 6z(t) = 3\delta(t-\epsilon), \quad z(0) = 0, \quad z'(0) = 4, \quad \epsilon > 0.}$$

   What happens as $\epsilon \longrightarrow 0^+$?

   (b) Find the general solution $y(x)$:

$$\mathbf{9x^2y''(x) + 15xy'(x) + y(x) = 0, \quad x > 0.}$$

4. (a) Classify all finite non-negative points of the ODE: (25 points total)

$$\mathbf{x^2y''(x) + xy'(x) + (x^2 - 1/16)y(x) = 0, \quad x > 0}$$

   and then give only the general form of the power series solution in the regular case (say about $x = a$) and the singular case, but do not evaluate any coefficients.

   (b) Find a power series approximation to the solution $y(x)$ of the IVP:

$$\mathbf{y''(x) + 3xy'(x) - 5y(x) = 0, \quad y(0) = 4, \quad y'(0) = 2,}$$

   by assuming an expansion about $x = 0$ of the form $\mathbf{y(x) \simeq a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5}$, finding the coefficients $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. You do not need any more terms!

Table of Laplace Transforms[1]

$\mathbf{f(t) = \mathcal{L}^{-1}(s)](t)}$	$\mathbf{F(s) = \mathcal{L}[f(t)](s)}$
$\mathbf{f(at)}$	$\mathbf{\frac{1}{a}F\left(\frac{1}{a}\right)}$
$\mathbf{e^{at}f(t)}$	$\mathbf{F(s-a)}$
$\mathbf{f^{(n)}(t)}$	$\mathbf{s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)}$
$\mathbf{t^nf(t)}$	$\mathbf{(-1)^nF^{(n)}(s)}$
$\mathbf{\frac{1}{t}f(t), f(0) = 0}$	$\mathbf{\int_s^\infty F(u)du}$
$\mathbf{\int_0^t f(v)dv}$	$\mathbf{F(s)/s}$
$\mathbf{(f * g)(t)}$	$\mathbf{F(s) \cdot G(s)}$
$\mathbf{f(t-a)u(t-a),\ a \geq 0}$	$\mathbf{e^{-as}F(s)}$
$\mathbf{g(t)u(t-a),\ a \geq 0}$	$\mathbf{e^{-as}\mathcal{L}[g(t+a)](s)}$
$\mathbf{e^{at}\sin(bt)}$	$\mathbf{\frac{b}{(s-a)^2+b^2}}$
$\mathbf{e^{at}\cos(bt)}$	$\mathbf{\frac{s-a}{(s-a)^2+b^2}}$
$\mathbf{\sinh(bt)}$	$\mathbf{\frac{b}{s^2-b^2}}$
$\mathbf{\cosh(bt)}$	$\mathbf{\frac{s}{s^2-b^2}}$

[1] From Back Cover of Nagle and Saff Text