MATH 220 **EXAM I** **10.9.98 - 12pm**

Directions: Answer all questions and show all work in the **exam booklet** provided. Start each new question at the **top** of a new page and **box** your final answer. Each question is worth 18 points.

1. (a) Solve the IVP:

$$\frac{dy}{dx} = \frac{e^{-y}}{y(x+1)}, \quad y(0) = 1$$

   (b) State Euler's algorithm for the equation in (a) using step size $h = 1/10$. You need not compute any values of y_n.

2. (a) Find the general solution of: $y'' + 8y' - 9y = 0$

   (b) Give the guess you would use to find a particular solution to: (*Do not evaluate the unknown constants*)

$$y'' + 4y' + 4y = x - 4 + xe^{-2x}$$

3. Consider a large tank holding 600 L of brine solution, initially containing 5 Kg of salt. At time $t = 0$, more brine solution begins to flow into the tank at the rate of 6 L/min. The solution inside the tank is well-stirred and is flowing out of the tank at the rate of 6 L/min. The concentration of salt in the solution entering the tank $te^{-t/100}$ Kg/L, i.e. varies in time. *Set up* and *solve* the problem for $A(t)$ = the amount of salt in the tank at time t.

4. Find the general solution $x(t)$ and $y(t)$ of the system

$$\begin{aligned} x' &= 3y, \\ y' &= 2x - y. \end{aligned}$$

5. Consider a mass-spring system with mass $m = 1$, damping constant $b = 2$, and spring constant $k = 5$. At $t = 0$, an external force $f(t) = 20\cos t$ is applied to the system. Find and sketch the "steady-state" solution for the displacement $x(t)$. Hint: The "steady-state" solution is the long time solution ignoring the the transient solution.

6. Find the general solution $y(x)$ - (You **must** use *variation of parameters* to find a particular solution - Hint: start with the formulas for v_1 and v_2)

$$y'' - y' - 2y = e^{3x}.$$