MATH 220 EXAM II

C. Tier — November 13, 1998

Answer all questions (25pts each) - **show all work in your exam booklet**.

1. (a) Find the general solution $y(x)$: $x^2y'' + 9xy' + 16y = 0, \quad x > 0$.

   (b) Find $h(t)$: $h'' + h' = \delta(t), \quad h(0) = 0, \quad h'(0) = 1$.

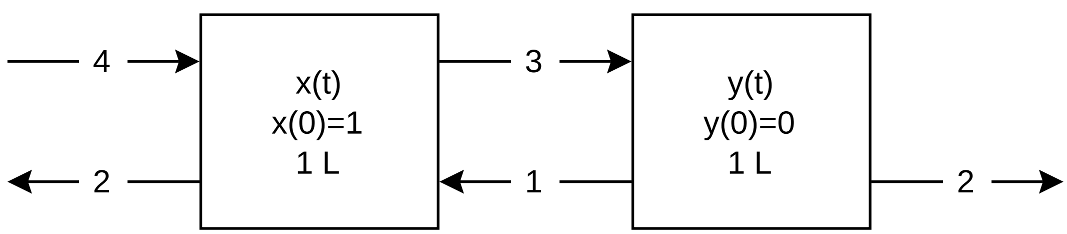
2. (a) $\mathcal{L}\{e^{-10t}t^4 + t\delta(t-2)\}$

   (b) $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2 + 2s + 10}\right\}$

   (c) $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2 + 9s + 8}\right\}$

3. (a) Find an approximation to the general solution of $y''+2xy'-4y = 0$ by assuming a solution of the form $y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$. You do not need any more terms!

   (b) Classify all points of the equation $x^2y'' + xy' + (x^2 - 1/4)y = 0$ and then state the general solution. Bonus: Explain whether or not all solutions of this equation are analytic at $x = 0$ by using the indicial equation for the ODE or from the functions in the general solution.



4. Consider a mixing problem between two interconnected tanks shown in the figure. Initially both tanks contain fresh water and the input into tank A is fresh water. At $t = 2$, a valve is switched and the fluid entering tank A (at rate 3 L/min) now has salt concentration 1 Kg/L (instead of the fresh water). Set up the model for the amount of salt in each tank and the find the Laplace transform for the amount of salt in tank B, $Y(s) = \mathcal{L}\{y(t)\}$. You need *not* invert the transform.

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
$f(at)$	$\frac{1}{a}F\left(\frac{1}{a}\right)$
$e^{at}f(t)$	$F(s-a)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^nf(t)$	$(-1)^nF^{(n)}(s)$
$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$
$\int_0^t f(v)dv$	$F(s)/s$
$f * g$	$F(s)G(s)$
$f(t-a)u(t-a),\ a \geq 0$	$e^{-as}F(s)$
$g(t)u(t-a),\ a \geq 0$	$e^{-as}\mathcal{L}\{g(t+a)\}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh bt$	$\frac{b}{s^2-b^2}$
$\cosh bt$	$\frac{s}{s^2-b^2}$