MATH 220 **Combined Final Exam** **6pm-8pm 06 May 1999**

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Directions: Answer **All Questions** and show **All Work** in the **Exam Booklet** provided. Write your **Name**, **Social Security Number**, and **Discussion Section Hour/Day** on the Exam Book Cover Page. Start each new question at the **top** of a new page and **box** your final answer. Each of 8 questions is worth 25 points. **Keep your eyes on your own work and keep your own work covered.** A *Table of Laplace Transforms and a table of Integrals are provided*, but must be returned at the end of the exam. You must also return your **Formulae Sheet** at the end.

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1. Obtain the solution $\mathbf{y}(\boldsymbol{x})$ to each of the following initial value problems:

   (a) $\mathrm{x}y'(x) + 2y(x) = x^2 - x + 1, \quad x > 0, \quad y(1) = \frac{3}{4}$

   (b) $\mathrm{y}'(x) = 2y^2 + xy^2, \quad y(0) = 1$

2. Find the general solution of

$$\mathrm{x}^2 y''(x) - 3xy'(x) + 4y(x) = x^2, \quad x > 0$$

3. Use the method of undetermined coefficients to find the the general solution of the ODE:

$$\mathrm{y}''(x) - 2y'(x) + y(x) = 3 + e^x + 2\sin(x)$$

4. Find all the values of the real number $\boldsymbol{\lambda} > 1$ for which the boundary value problem

$$\begin{array}{ll} \textbf{ODE :} & y''(x) + 2y'(x) + \lambda y(x) = 0, \quad 0 \le x \le 1 \\ & \textbf{BC:}\ \ y(0) = 0, \quad y(1) = 0 \end{array}$$

   has nontrivial solutions, $\mathbf{y}(\boldsymbol{x})$, and find these solutions.

5. Evaluate:

   (a) $\mathcal{L}\left[e^{-3\mathrm{t}}\sin(4t) + (t+2)^2 e^{-4\mathrm{t}}\right](s)$

   (b) $\mathcal{L}^{-1}\left[\frac{\mathrm{s}}{(\mathrm{s}+2)^2} + \frac{3\mathrm{s}+13}{\mathrm{s}^2+6\mathrm{s}+25}\right](t)$

6. Find the Laplace Transform of the solutions, $\mathbf{X}(\boldsymbol{s}) = \boldsymbol{\mathcal{L}}[\boldsymbol{x}(\boldsymbol{t})]$ and $\mathbf{Y}(\boldsymbol{s}) = \boldsymbol{\mathcal{L}}[\boldsymbol{y}(\boldsymbol{t})]$, satisfying the system of ODEs,

$$\mathrm{x}'(t) + 2y(t) = 3e^{\mathrm{t}}\cos(t)\delta(t-4), \quad x(0) = 5,$$
$$x(t) - y'(t) = 8u(t-4), \quad y(0) = 0.$$

(Solve, but do not simplify.)

7. Some partial PDE problems:

   (a) Apply the method of separation of variables to the PDE ("Wave Equation"),

$$\text{PDE}: \frac{\partial^2 u}{\partial t^2}(x,t) = 3\frac{\partial^2 u}{\partial x^2}(x,t), \quad 0 < x < 8, \quad t > 0;$$
$$\text{BC:} \quad u(0,t) = 0 = u(8,t), \quad t > 0,$$

   and obtain the two ODEs that the function of $\mathbf{x}$ and function of $\mathbf{t}$ must satisfy. Also determine the BC that the function of $\mathbf{x}$ must satisfy (do not solve).

   (b) Find the steady state solution only for the inhomogeneous PDE (heat equation) problem,

$$\text{PDE}: \frac{\partial u}{\partial t}(x,t) = 3\frac{\partial^2 u}{\partial x^2}(x,t) - 9e^{\mathrm{X}}, \quad 0 < x < 8, \quad t > 0;$$
$$\text{BC:} \quad u(0,t) = 30, \quad u(8,t) = 70, \quad t > 0,$$

8. (a) Find the Fourier series for the function,

$$\mathrm{f}(x) = \left\{ \begin{array}{ll} 0, & -\pi < x < -\pi/2 \\ 4, & -\pi/2 < x < +\pi/2 \\ 0, & +\pi/2 < x < +\pi \end{array} \right\},$$

   by computing the Fourier coefficients.

   (b) Graph the extended function to which the Fourier series converges in the interval $[-2\pi, +2\pi]$ and be sure to include the values at the points of discontinuity.