Math 574 Applied Optimal Control – Hanson – Fall 2006

(Stochastic Processes and Control for Jump-Diffusions)

Homework 3 – Stochastic Jump Integration
(See Chapter 3 of Text; See also Chapter 0 for Preliminaries)

Homework due 18 October 2006 in class.

Correction, 12 October 2006

1. Show that the partial sums of the **geometric series** can be summed as

$$S_n \equiv \sum_{k=0}^{n} x^k = \left\{ \begin{array}{cc} \dfrac{1-x^{n+1}}{1-x}, & x \neq 1 \\ n+1, & x = 1 \end{array} \right\}, \tag{1}$$

for integers $n \geq 0$ by showing that the difference of the defined summation (ΔS_n) and the difference of the summed answer to the far right are the same and that the discrete initial conditions are the same at $n = 0$.

2. Show that

$$\int_0^t e^{aP(s)} dP(s) = \left\{ \begin{array}{cc} \dfrac{e^{aP(t)}-1}{e^a - 1}, & a \neq 0 \\ P(t), & a = 0 \end{array} \right\}, \tag{2}$$

for real constant a, in two ways, showing that they give the same answers,

   (a) Using the pure Poisson sum form $\sum_{k=0}^{P(t)-1} h(k)$ for function h of Theorem 3.8 and the geometric series partial sum results in (1) of Problem 1.

   (b) Using the Zero-One Jump Law and the Fundamental Theorem of Jump Calculus applied to $d\exp(aP(t))$ to evaluate the integral.

3. Show that the power rules for stochastic integration for Poisson noise can be written as the recursions,

$$\int_0^t P^m(s)dP(s) = \frac{1}{m+1}\left(P^{m+1}(t) - \sum_{k=2}^{m+1} \binom{m+1}{k} \int_0^t P^{m+1-k}(s)dP(s)\right), \tag{3}$$

using the jump form of the stochastic chain rule and mathematical induction.

   (a) Illustrate the application of the formulae for $P(t)$ to find the results for the cases $m = 2$ and $m = 3$, to get explicit formulas that do not have an integral. See Table 3.1.

4. Show the mean square limit for the product of $dP(t)$ and $dW(t)$ in (3.17-3.18) by proving that

$$\mathrm{Var}\left[\sum_{i=0}^{n} \Delta P_i \Delta W_i\right] \to 0, \tag{4}$$

as $n \to +\infty$ and $\delta t_n \to 0^+$.