# Math 574 Applied Optimal Control – Hanson – Fall 2006

## (Stochastic Processes and Control for Jump-Diffusions)

## Homework 5 – General SDEs (Chapters 5 of Text; See also a small part of Chapter 11 and Chapter 0 for Preliminaries)

*Homework due 20 November 2006 in class.*
*Acknowledge consultation with others, else receive a grade discount.*

1. Simulate $X(t)$ for the log-normally distributed jump amplitude case with mean $\mu_j = \mathrm{E}[Q] = 2.74$ and variance $\sigma_j^2 = \mathrm{Var}[;Q] = 1.38$ for the linear jump-diffusion SDE model for a population in a seasonal environment $\mu_d(t) = 0.1085\sin(2\pi t - 0.75\pi)$, $\sigma_d(t) = 0.0485 - 0.0233\sin(2\pi t - 0.75\pi)$, $\lambda(t) = 3.98 - 0.0115\sin(2\pi t - 0.75\pi)$ per year, $\nu(t,Q) = \exp(Q) - 1$ with $Q$ normally distributed, $X(0) = 0.5$, $t_0 = 0$, $t_f = 3.0$ years, $N + 1 = 10,000$ sample points per state for M $= 4$ states.

   {*Hint: See the linear mark-jump-diffusion SDE simulator Example 5.25 with Appendix A MATLAB code A.15.* }

2. For the log-double-uniform jump distribution,

$$\phi_Q(q;t) \equiv \left\{ \begin{array}{ll} 0, & -\infty < q < a(t) \\ p_1(t)/|a|(t), & a(t) \le q < 0 \\ p_2(t)/b(t), & 0 \le q \le b(t) \\ 0, & b(t) < q < +\infty \end{array} \right\} , \tag{1}$$

   where $p_1(t)$ is the probability of a negative jump and $p_2(t)$ is the probability of a posative jump on $a(t) < 0 \le b(t)$, show that

   (a) $\mathrm{E}_Q[Q] = \mu_j(t) = (p_1(t)a(t) + p_2(t)b(t))/2$;

   (b) $\mathrm{Var}_Q[Q] = \sigma_j^2(t) = (p_1(t)a^2(t) + p_2(t)b^2(t))/3 - \mu_j^2(t)$;

   (c) $E_Q[(Q - \mu_j(t))^3] = (p_1(t)a^3(t) + p_2(t)b^3(t))/4 - \mu_j(t)(3\sigma_j^2(t) + \mu_j^2(t))$;

   (d) $\mathrm{E}[\nu(Q)] = \mathrm{E}[\exp(Q) - 1]$, where the answer needs to be computed.

3. Show that the Itô mean square limit for correlated bond-stock price noise at time $t$ (11.16)

$$dW_B(t)dW_S(t) \overset{ims}{=} \rho(t)dt , \tag{2}$$

   is valid, where

$$\mathrm{Cov}[\Delta W_B(t_i), \Delta W_S(t_i)[] \simeq \rho(t_i)\Delta t_i$$

   for sufficiently small $\Delta t_i$ and

$$\int_0^t |\rho|(s)ds < \infty.$$

   Are there any special treatments required if $\rho = 0$ or $\rho = \pm 1$? You may use the bivariate normal density in (0.136) or Table 0.1 of selected moments of preliminaries Chapter 0.