Math 574 Applied Optimal Control – Hanson – Fall 2006

(Stochastic Processes and Control for Jump-Diffusions)

Homework 6 – Deterministic Control (Chapters 6; See also Chapter 0 for Preliminaries)

Homework due 29 November 2006 in class.
Acknowledge consultation with others, else receive a grade discount.

1. Consider another simple lumped model of a leaky reservoir given by

$$\dot{X}(t) = -aX(t) + U(t), \quad X(0) = x_0,$$

where $X(t)$ is the depth of the reservoir, $U(t)$ is the net flow of water per unit time into the reservoir at time t constrained and $a > 0$ is the rate of leakage and usage. The net inflow is constrained pointwise $0 \le U(t) \le M$ for all $0 < t \le t_f$ and also cumulatively by

$$\int_0^{t_f} U(t)dt = K > 0,$$

where K, M and t_f are fixed constants, such that $K \le M \cdot t_f$ for consistency. Find the optimal control law that maximizes only the final depth,

$$J[X] = bX(t_f)$$

with $b > 0$ and the optimal state $X^*(t)$.

2. For the deterministic linear first order dynamics,

$$\dot{X}(t) = -\mu_0 X(t) + \beta_0 U(t), t > 0, \;\text{ given } X(0) = x_0 \ne 0, \;\; \mu_0 > 0, \;\; \beta_0 \ne 0,$$

and quadratic performance measure,

$$V[U] = \frac{r_0}{2}\int_0^{t_f} U^2(t)dt, \quad r_0 > 0,$$

find the optimal state trajectory and optimal (unconstrained) control to bring the state from the initial state to the final state x_f in t_f seconds while minimizing the functional $V[U]$ with respect to the control u, with the answer depending on the parameter set $\{x_0, x_f, t_f, \mu_0, \beta_0, r_0\}$. *Note that the final state and time iare fixed.*