Math 586 Computational Finance – Hanson – Spring 2008

Homework 1 – Compound Interest and Stochastic Differential Equations

- *Homework due 06 February 2008 in class.*
- *Justify all steps by supplying the reason(s).*
- *This is individual homework: you may discuss generally with others if cited, but final submission must be your own work.*
- MATLAB, Maple or Mathematica solutions are acceptable, if appropriate.

1. (a) Compute the *future value* of a \$1000 investment in a money market fund with interest compounded *quarterly* at 4.5% annual interest over 5 years.
   (b) Compute the *present value* of a future balance of \$1000 at the end of 5 years at 4.5% annual interest compounded *quarterly*.

2. (a) Compute the *future value* of a \$1000 investment in a money market fund with interest compounded *continuously* at 4.5% annual interest over 5 years. Also, compute the percentage difference between the the future value obtained in problem 1 with the future value obtained in this part.
   (b) Compute the *present value* of a future balance of \$1000 at the end of 5 years at 4.5% annual interest compounded *continuously*. Also, compute the percentage difference between the the present value obtained in problem 1 with the present value obtained in this part.

3. Compute the *present value* of a cash stream at the end of 5 years compounded *continuously*, but with a *variable* annual interest rate of 4.5% the first year, 4.75% the second, 5.25% the third, 5.75% the fourth and 5.50% the fifth, final year. The continuous cash stream has the functional form $P(t) = \$1000 \cdot \exp(-0.025 \cdot t)$ for $t \in [0, 5.0]$ years.

4. For the diffusion process differential:
   (a) Show that $\mathrm{E}[(dW)^3(t)] = 0$, exactly.
   (b) Show that $\mathrm{E}[(dW)^4(t)] = 3(dt)^2$, exactly.

5. Simulate the solution to the following variable coefficient diffusion SDE:

$$dS(t) = S(t) \cdot (\mu(t) \cdot dt + \sigma(t) \cdot dW(t)),$$

   where $S(0) = \$1000$, $\mu(t) = 0.20 \cdot (1 + t)/(1 + 2 \cdot t)$, and $\sigma(t) = 0.30 \cdot (1 + 2 \cdot t)/(1 + 3 \cdot t)$ on [0,2] years, using $N = 4000$ time steps. Also, plot the simulated trajectory with proper title and axis labels. Include your code or worksheet. (For sample MATLAB code, see `http://www.math.uic.edu/~hanson/math586/Class08Codes/linear_diffusion08sims.m`, "Linear-Diffusion Stock Price, S(t)", or see Math 586 Spring 2008 Codes on class homepage.