**Math 586 Computational Finance – Hanson – Spring 2008**

**Homework 2 – More Stochastic Differential Equations and Stock Options**

- *Homework due 07 March 2008 in class.*
- *For computations, round final results to 4 significant digits (e.g., 12.34 or 0.01234 or 1.234*$\cdot 10^4$*), except round monetary values to the nearest hundredths of a dollar.*
- *This is individual homework: you may discuss generally with others if cited, but final submission must be your own work.*
- MATLAB computational solutions are recommended; Maple or Mathematica symbolic proofs or solutions are acceptable, if appropriate; you can ask Prof. Hanson for MATLAB help.

*See correction(s) in red, 03 March 2008.*

1. Computationally confirm the **class** $dt$**-precision multiplication table** by simulating just the 4 integral representations:

$$\int_0^t dx, \quad \int_0^t (dW)^2(x), \quad \int_0^t (dW)^3(x), \quad \text{and} \quad \int_0^t dx dW(x)$$

by forward sums with $N = 2000$ on $0 \le t \le T = 2.0$. Plot the results versus $t$. Also, compute the mean errors, mean$(E_j)$, and standard deviations, std$(E_j)$, for processes $j = 1:4$, where $E_j = [S_j(t_i) - L_j(t_i)]_{1\times(N+1)}$ is the vector difference between the sum approximations $S_j$ for the $j$th process and $L_j$ is the exact Itô $dt$-precision value, while $t_i = (i-1)\cdot\Delta t$ for $i = 1:N+1$. {*See MATLAB help for the built-in functions* `mean` *and* `std`*. Your code for Question #5 of Homework #1 can be simplified and revised if desired; else go back to the original code:* `http://www.math.uic.edu/~hanson/math586/Class08Codes/linear_diffusion08sims.m`}

2. Computationally confirm the **convergence rate difference** between the direct simulation of the S$\Delta$E,

$$S(t + \Delta t) = S(t) \cdot (1 + \mu(t)\Delta t + \sigma(t)\Delta W(t)),$$

and the forward approximation in the exponent of the exact exponential solution,

$$S(t + \Delta t) = S(t) \cdot \exp((\mu(t) - \sigma^2(t)/2)\Delta t + \sigma(t)\Delta W(t)),$$

for the $i = 1:4$ samples with $N = [100, 1000, 10000, 100000]' = (10^{(1+1:4)})'$, where the symbol **{'}** denotes the matrix transpose in *MATLABese* and helps here to nicely line up subscripts. For comparison and efficiency, initially generate {*i.e., use MATLAB's* `randn` *prior to scaling*} one set of standard normal variables $[Z(1,j)]_{1\times N(4,1)}$ for $i = 4$, the largest sample corresponding to the times $[t(4,j)]_{1\times N(4,1)}$, where

$$t(i,j) \equiv (j-1)\cdot\Delta t(i,1) \text{ for } j = 1:N(i,1)+1 \quad \text{and} \quad \Delta t(i,1) \equiv T/N(i,1) \text{ for } i = 1:4,$$

such that $t(i, j_i) = t(4, j_4(i))$ for $i = 1:4$, so

$$j_4(i) \equiv (j_i - 1)\cdot\Delta t(i,1)/\Delta t(4,1) + 1 = (j_i - 1)\cdot N(4,1)/N(i,1) + 1, \text{ for } j_i = 1:N(i,1)+1.$$

Assume $S_0 = \$100$, $\mathbf{T = 2,}$ $\mu(t) = 0.23\cdot(1 + t/10)$ and $\sigma(t) = 0.36\cdot(1 + t/10)$.
Present results as the standard deviation (*MATLAB's* `std`) for each $N(i,1)$ of

(a) the difference between the S$\Delta$E and the exact approximations and

(b) the errors in both the SΔE and the exact approximations, for each $N(1:3,1)$, assuming that the $N(4,1)$ exact approximation, using the common set of time values for each $N(1:3,1)$, can be substituted for an exact solution.

{*Caution: for row or column vectors,* `std` *returns the unbiased estimate of the standard deviation, i.e., averaging with* $N-1$ *instead of* $N$ *and this is appropriate for the simulation samples here, but for non-vector matrices it returns the standard deviations of the columns as a row vector.*}

3. Consider a **Bull Call Spread** position, for example, to

   (a) long (buy) a call option for \$6.50 with an exercise price of $K_{\text{buy}} = \$85$, and

   (b) short (sell) another call option for \$3.50 on the same underlying stock with an exercise price of $K_{\text{sell}} = \$96$, each for a common exercise date of $T = 6$ months.

   (c) Discuss the relationship between exercise prices and between option prices needed for this position to work.

   For this spread:

   (a) Derive the general formula with general variables for the final value of the position at $T$.

   (b) Clearly present the net profit or loss graph versus the $S(T)$ value for the above numerical example, illustrating the constituent calls that make up the spread and mark the break-even point, if any.

   (c) Discuss the relationship between exercise prices and between option prices needed for this position to work.

4. Consider a **Bear Put Spread** position to, for example,

   (a) buy a put option for \$7.50 with an exercise price of $K_{\text{buy}} = \$75$, and

   (b) simultaneously another put option for \$4.50 on the same underlying stock and at the same exercise date of $T = 6$ months to sell with an exercise price of $K_{\text{sell}} = \$60$.

   For the bear put spread:

   (a) Derive the general formula with general variables for the final value of the position at $T$.

   (b) Clearly present the net profit or loss graph versus the $S(T)$ value for the above numerical example, illustrating the constituent calls that make up the spread and mark the break-even point, if any.

   (c) Discuss the relationship between exercise prices and between option prices needed for this position to work.

5. Consider an option combination called a **Straddle** to, for example,

   (a) long (buy) a call option for \$3.50 with an exercise price of $K_{\text{buy}} = \$93$, and

   (b) long (buy) a put option for \$7.50 to sell the same underlying stock with an exercise price of $K_{\text{sell}} = \$93$, each for a common exercise date of $T = 6$ months.

   For this spread:

   (a) Derive the general formula with general variables for the final value of the position at $T$.

(b) Clearly present the net profit or loss graph versus the $S(T)$ value for the above example, illustrating the constituent calls that make up the spread and mark the break-even point, if any.

(c) Discuss the relationship between exercise prices and between option prices needed for this position to work.