Math 586 Computational Finance – Hanson – Spring 2008

Homework 3 – Black-Scholes and Merton Models; European Option Pricing; Simple American Option Approximations; Dividends

- ***Homework now due 07 April 2008 in class.***
- *For computations, round final results to 4 significant digits (e.g., 12.34 or 0.01234 or* $1.234 \cdot 10^4$*), except round monetary values to the nearest hundredths of a dollar; use MATLAB built-in double-precision for intermediate calculations; hand in documented code with documented output.*
- *This is individual homework: you may discuss generally with others if cited, but final submission must be your own work. Also, cite any outside sources or references that you use. This includes outside code sources, which also have to be verified for correctness and accuracy.*
- *MATLAB computational solutions are recommended; Maple or Mathematica symbolic proofs or solutions are acceptable, if appropriate; you can ask Prof. Hanson for MATLAB help.*

See correction(s) in red, 01 April 2008.

1. **Black-Scholes European Option Pricing:** Let $S_0 = \$100$, $r_0 = 5.75\%$ per year (p.a.) and $\sigma_0 = 18\%$ without dividends.

   (a) Compute the Black-Scholes call prices for strike prices $K = 80 : 5 : 120$ in US dollars for each exercise time $T = 0.25 : 0.25 : 1.00$ years.

   (b) Similarly, compute the European put price directly from Black-Scholes pricing for puts.

   (c) Plot the call prices versus the strike prices K with T-values as the parameter for each respective curve using different symbols or other distinct markings.

   (d) Separately plot the put prices similarly.

   (e) Verify the Put-Call Parity using the Black-Scholes put and call prices, plotting the percentage errors relative to the Black-Scholes put prices versus K and parameterized by T on one plot.

   {*Comments:*

   - *You may code your own programs, or use modifications of the Global Derivatives Black-Scholes MATLAB code, as long as you verify it,*
     `http://www.global-derivatives.com/code/matlab/BlackScholesEuro.m`
     with instructions in the m-code preface and further explanations in
     `http://www.global-derivatives.com/index.php?option=com_content&task=view&id=52&Itemid=31`
   - *If your version of MATLAB has the normcdf, then you may use it, else use the* `erfc` *function relation given in class or by MATLAB help.*
   - *It is suggested that when your main MATLAB m-code calls other functions, you can avoid path and structure problems by naming your main program a function and copy-pasting all available called functions at the end of main so that they are proper subfunctions (name is still function).*

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2. **Merton's European Option Pricing with Constant Yield Dividend:** Let $S_0 = \$100$, $r_0 = 10.0\%$ p.a., $\sigma_0 = 18\%$ and $T = 1.0$years with a single known dividend yield D.

   (a) Compute the Black-Scholes call prices for strike prices $K = 80 : 5 : 120$ in US dollars for each known dividend yield $D = 2 : 2 : 8$ percent per year.

   (b) Similarly, compute the European put price directly from Black-Scholes pricing for puts.

   (c) Plot the call prices versus the strike prices K with D-values as the parameter for each respective curve using different symbols or other distinct markings.

   (d) Separately plot the put prices similarly.

   (e) Discuss the effects of the constant dividend yield and compare the results to the non-dividend case in the first problem.

   {*Comments:*

   - *You may code your own programs, or use modifications of the Global Derivatives Black-Scholes Merton MATLAB code, as long as you verify it,* `http://www.global-derivatives.com/code/matlab/BlackScholesMertonEuro.m` *with instructions in the m-code preface (this code is a generalization of the code mentioned in the first question, but lacks minor polish, e.g., the null output argument "[ ]" needs to be replaced by actual arguments like "[CallPrice, PutPrice]" and some missing semi-colons should be inserted) and further explanations in* `http://www.global-derivatives.com/index.php?option=com_content&task=view&id=52&Itemid=31`
   - *As in the first problem, if your version of MATLAB has the normcdf, then you may use it, else use the* `erfc` *function relation given in class or by MATLAB help.*
   - *Again, it is suggested that when your main MATLAB m-code calls other functions, you can avoid path and structure problems by naming your main program a function and copy-pasting all available called functions at the end of main so that they are proper subfunctions (subfunction is still a function).*

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3. **RWG and Black Approximations for American Call Option Prices with Early Exercise prior to the Final Discrete Dividend:** Let $S_0 = \$100$, $r_0 = 6.0\%$ p.a., $\sigma_0 = 15\%$ and $T = 1.0$ years with a known final dividend amount $D1$ at date $T1 = 8/12$ years.

   (a) Compute the RGW early-exercise American call prices for strike prices $K = 80 : 5 : 120$ in US dollars for each known final dividend amounts $D1 = 1 : 0.5 : 3$ in US dollars per share.

   (b) Similarly, compute Fischer Black's approximation to early-exercise American call prices for strike prices $K = 80 : 5 : 120$ in US dollars for each known final dividend amounts $D1 = 1 : 0.5 : 3$ in US dollars per share according to Hull and class discussion.

   (c) Plot the RGW approximation to American call prices versus the strike prices K with $D1$-values as the parameter for each respective curve using different symbols or other distinct markings.

   (d) Similarly and separately plot Black's approximation the American call prices.

   (e) Discuss the effects of the dividend payout and compare the results to the non-dividend case in the first problem.

   {*Comments:*

- *You may code your own programs, or use modifications of the code zipped-folder RGW.zip on the Global Derivatives MATLAB code page, as long as you verify it,*
  http://www.global-derivatives.com/index.php?option=com_content&task=view&id=184&Itemid=41
  or you can access the unzipped folder at
  http://www.math.uic.edu/~hanson/math586/Class08Codes/RGW
  with instructions in the m-code rogewhaley.m and further explanations in
  http://www.global-derivatives.com/index.php?option=com_content&task=view&id=14
  as well as in Hull's Technical Notes 4 and 5.
- *See the Readme.m for information and rogewhaley.m and bsprice.m function files.*
- *This code has its own normcdf m-file called normcdfM, which may be usable in the prior problems.*
- *Again, it is suggested that when your main MATLAB m-code calls other functions, you can avoid path and structure problems by naming your main program a function and copy-pasting all available called functions at the end of main so that they are proper subfunctions (subfunction is still a function).*

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4. **The Greeks:** From the Black-Scholes formula, calculate:

   (a) The deltas of both calls and puts, i.e.,

   $$\Delta_C = \frac{\partial C}{\partial s}(s,t) \ \text{ and } \ \Delta_P = \frac{\partial P}{\partial s}(s,t),$$

   obviously by differentiation.

   (b) The vegas of both calls and puts, i.e.,

   $$\mathcal{V}_C = \frac{\partial C}{\partial \sigma_0}(s,t;K,T,r_0,\sigma_0) \ \text{ and } \ \mathcal{V}_P = \frac{\partial P}{\partial \sigma_0}(s,t;K,T,r_0,\sigma_0).$$