## MCS471 — Spring99 — Numerical Analysis — Exam1 — Hanson FIRST TRITERM EXAM - Precision & Nonlinear Equations

2pm — 15 February 99 — 316 Burnham Hall

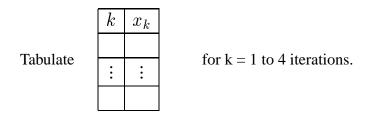
**THE RULES**: In computational questions, use *4 Digit Exam Precision*: **Chop to 4** <u>significant</u> decimal digits only when you record an intermediate or final answer in your exam booklet; and continue calculations with these chopped, recorded numbers. Note the tables displayed here are examples only, the number of rows needed in your exam booklet may vary.

0.1	State your calculator's	
	a) Brand,	c) Full Precision without exponential notation mode,
	b) Model,	d) Is it Programmable?
		e) Also, Undergraduate or Graduate Student?

1. Find a numerical approximation to the intersection between two functions

$$e^{x/4} = 1.982/x \tag{1}$$

starting from  $x_1 = 1.300$  at k = 1 by forming a <u>convergent</u> Fixed Point Iteration, other than Newton's method. Demonstrate that the fixed point convergence criterion is satisfied near  $x_1$ .



2. Using Newton's Method, find a numerical approximation to the Zero of

$$f(x) = e^{x/4} - 2.512/x \tag{2}$$

on [1.0,2.2] starting at k = 1 iterations ( $k_{fe}$  is the number of function evaluations) with the endpoint having the smallest value of |f|, keeping track of the current change in sign interval ( $a_k, b_k$ ), tabulating

k	$k_{fe}$	$a_k$	$b_k$	$x_k$	$f_k$	$f'_k$	$x_{k+1}$	$ \Delta x_k $

until  $|\Delta x_k| = |x_{k+1} - x_k| < 0.5e - 2.$ 

3. Using the method of Golden Section Search, find the Maximum and its Interval of Uncertainty for

$$g(x) = x * (3 - e^{x/4})$$
(3)

on [2.1,2.9]. Use the rounded version of the GSS constant  $c \simeq 0.3820$ . Summarize your results with a table of

k	$k_{fe}$	$a_k$	$b_k$	$d_k$	$x_k$	$u_k$	$gx_k$	$gu_k$

for k = 1 to 3 iterations with  $k_{fe}$  the number of function evaluations. Circle and label your best approximation to the maximum, its location, and state the interval of uncertainty.

4. Show how to find the <u>Maximum</u> of the function in Question 3 using Newton's Method to find the critical point of the function (hint: by using the derivative in place of the function value), computing the one Newton iterate for the location of the critical point and the value of the function g(x) at that iterate, starting at x = 2.500.

**points per question vector** =  $\begin{bmatrix} 1 & 25 & 25 & 24 \end{bmatrix}^T$