MCS471 — Spring99 — Numerical Analysis — Exam1 — Hanson

FIRST TRITERM EXAM - Precision & Nonlinear Equations

2pm — 15 February 99 — 316 Burnham Hall

THE RULES: In computational questions, use *4 Digit Exam Precision*: **Chop to 4 significant decimal digits** only when you record an intermediate or final answer in your exam booklet; and continue calculations with these chopped, recorded numbers. Note the tables displayed here are examples only, the number of rows needed in your exam booklet may vary.

0.1 State your calculator's

- a) Brand,
- b) Model,
- c) Full Precision without exponential notation mode,
- d) Is it Programmable?
- e) Also, Undergraduate or Graduate Student?

---

1. Find a numerical approximation to the intersection between two functions

$$e^{x/4} = 1.982/x \tag{1}$$

starting from $x_1 = 1.300$ at $k = 1$ by forming a *convergent* **Fixed Point Iteration**, other than Newton's method. Demonstrate that the fixed point convergence criterion is satisfied near x_1.

Tabulate

k	x_k
$\vdots$	$\vdots$

for k = 1 to 4 iterations.

2. Using **Newton's Method**, find a numerical approximation to the **Zero** of

$$f(x) = e^{x/4} - 2.512/x \tag{2}$$

on [1.0,2.2] starting at $k = 1$ iterations (k_{fe} is the number of function evaluations) with the endpoint having the smallest value of $|f|$, keeping track of the current change in sign interval (a_k, b_k), tabulating

k	k_{fe}	a_k	b_k	x_k	f_k	f'_k	x_{k+1}	$\|\Delta x_k\|$

until $|\Delta x_k| = |x_{k+1} - x_k| < 0.5e-2$.

More on next page $\longrightarrow$.

3. Using the method of **Golden Section Search**, find the **<u>Maximum</u>** and its **Interval of Uncertainty** for

$$g(x) = x * (3 - e^{x/4}) \tag{3}$$

on [2.1,2.9]. Use the rounded version of the *GSS* constant $c \simeq 0.3820$. Summarize your results with a table of

k	k_{fe}	a_k	b_k	d_k	x_k	u_k	gx_k	gu_k

for k = 1 to 3 iterations with k_{fe} the number of function evaluations. Circle and label your best approximation to the maximum, its location, and state the interval of uncertainty.

4. Show how to find the **<u>Maximum</u>** of the function in Question 3 using **Newton's Method** to find the **critical point** of the function (hint: by using the derivative in place of the function value), computing the one Newton iterate for the location of the critical point and the value of the function $g(x)$ at that iterate, starting at $x = 2.500$.

---

points per question vector $= [1 \;\; 25 \;\; 25 \;\; 25 \;\; 24]^T$