MMS Production Scheduling Subject to Strikes in Random Environments

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Abstract

A multistage manufacturing system (MMS) is the normal paradigm used for the final assembly of a consumable good, for example a car or printed circuit board. A simple MMS consists normally of a linear sequence of workstations at which components or value is added to a product, this is essentially an assembly line. When planning the production scheduling, normally only workstation repair, failure, and defective pieces are considered as stochastic events that can affect the production rates for the various workstations. Additionally, it is assumed that all of the raw materials necessary to assemble the finished product are available, and therefore the supply routing problem of raw materials is not considered. In this treatment of the problem, we consider the effects of strikes and of natural disasters. A strike or natural disaster can affect the MMS itself and/or the way in which raw materials are introduced into the MMS that is the supply routing problem is treated as well. A numerical example is presented to illustrate the model that includes a strike as well as workstation repair and failure.

1. Introduction

The final assembly of a consumable good can be modeled by a multistage manufacturing system (MMS). In this manufacturing system, raw materials in the form of component parts are input into the system at various stages in order to produce the finished product. In determining the production rates for the various workstations in each stage, we have previously only considered workstation repair, failure, and small fluctuations due to defective pieces in the control model [11, 12, 14].

In these treatments it is assumed that the raw materials are present when they are needed. That is the *loading stage*, the mechanisms by which raw materials enter the MMS, and the *unloading or final stage*, the mechanisms by which the finished goods are delivered, are not considered as a part of these models. This is an essential assumption in order to determine the production rates, from these production rates the demand of raw materials for the various stages can then be determined. This determination of a timetable for the arrival of the raw components allows for the system to utilize a *Just in Time* or *stockless production* manufacturing discipline (see Hall [4]). *Just in Time* manufacturing systems are designed so that large inventories of raw materials are not necessary, the goal is to keep the quantity on hand of raw materials to just meet the desired production goal over the time horizon of the production run. Furthermore, Bielecki and Kumar [2], show that for an unreliable manufacturing system, the optimal policy is a zero inventory policy. This timetable must allow for all aspects of the deliveries of the raw materials to arrive in a timely manner.

The *routing* of raw materials into the MMS is a very difficult problem. This is one of the key features that differentiate the local perspective of a flexible manufacturing system (FMS), from that of the global view of a MMS (see Kimemia and Gershwin [7]). In a FMS, each piece needs to be tracked and routed to all of the workstations necessary to complete it, whereas in the MMS the focus is on the global process that is more concerned with the throughput of the system instead of tracking/routing individual pieces. Note, that each stage of a MMS can be viewed as a FMS.

For the MMS formulations considered in this paper, the true routing problem is not considered, but catastrophic events that affect the delivery of the raw materials are considered. We consider two types of strikes, primary and secondary. Primary strikes directly affect either the production of components, raw materials, or the actual labor used in the assembly process of the MMS. This is motivated by the strikes against the automobile manufacturer General Motors (GM) in 1996 and 1998 [1, 6, 10]. In both cases, a small number of employees went on strike that produced essential parts necessary for the final assembly of cars. Due to the lack of raw materials for the production of cars the assembly lines (MMS)

*Work supported in part by the National Science Foundation Grants DMS-96-26692 and DMS-99-73231. Preprint of paper published in *Proceedings of 2000 American Control Conference*, pp. 2194-2198, 28 June 2000.

were idled. Since GM utilizes a *Just in Time* manufacturing discipline, the work stoppage at the striking plants idled all of the workers in all of the plants producing the raw materials as well as those who assembled the automobiles. In both cases, GM suffered great financial losses, with total losses for the two month 1998 strikes reported as $2 billion so that the current climate between the manufacturer GM and UAW (United Auto Workers) union being less belligerent [3]. Secondary strikes are strikes that do not directly affect the MMS itself, but affect the delivery of raw materials to be used in the MMS. In today's manufacturing environment, it is not uncommon to rely upon raw materials that may come from great distances and foreign countries. Thus transportation of the raw materials to the manufacturing site is of great importance, especially for *Just in Time* manufacturing disciplines. The inclusion of this feature is motivated by the United Parcel Service (UPS) strike in 1997. In this strike, over 65% of the workers went on strike, which reduced the delivery capacity to about 30%. This had a wide impact on the United States since UPS is responsible for delivering more than 80% of all parcels [8, 9].

Natural disasters can occur that can affect the MMS in a wide variety of ways. These types of catastrophic events can directly affect either the MMS itself, the production of raw materials/components, or the delivery infrastructure. For an example of the application of natural disasters to optimal resource harvesting with price fluctuations see Hanson and Ryan [5]. Mass mortalities, severe weather, or governmental regulation in the harvesting of a consumable food item, for example salmon, can have a great impact on the fisheries industry whose canning process can be viewed as a MMS.

The reduction of the assumption that the raw materials are present when needed to accounting for delays in the deliveries of materials for various reasons allows for additional realism into the planning of a production run for a consumable good. The final assembly of an automobile or a printed circuit board in a MMS provides excellent motivation to study this problem. In this paper, we consider the effects of strikes and natural disasters to the routing portion of the raw materials for the MMS in conjunction with workstation repair, failure, and small fluctuations due to defective pieces in the control model. One of the goals of this production-scheduling problem is to reduce the impact of the rare events of strikes and natural disasters on the production of the final consumable good.

In Section 2., a linear dynamical formulation (LQGP Problem see [11]) of the MMS is presented and in Section 3. a numerical example.

2. LQGP Problem Formulation for MMS

The model presented here is in the canonical form for the LQGP problem that originally appears in Westman and Hanson [11] with modifications for state dependent Poisson noise [13]. The linear dynamical system for the LQGP problem is governed by the stochastic differential equation (SDE) subject to Gaussian and state dependent Poisson noise disturbances is given by

$$\begin{aligned} d\mathbf{X} &= [A\mathbf{X} + B\mathbf{U} + \mathbf{C}]dt + Gd\mathbf{W} \\ &+ [H_1 \cdot \mathbf{X}]d\mathbf{P}_1(\mathbf{X},t) + [H_2 \cdot \mathbf{U}]d\mathbf{P}_2(\mathbf{X},t) \\ &+ H_3 d\mathbf{P}_3(\mathbf{X},t), \end{aligned} \quad (1)$$

for general Markov processes in continuous time, with $m \times 1$ state vector $\mathbf{X}$(t), $n \times 1$ control vector $\mathbf{U}$(t), $r \times 1$ Gaussian noise vector d$\mathbf{W}$(t), and $q_\ell \times 1$ space-time Poisson noise vectors $d\mathbf{P}_\ell(\mathbf{X}(t),t)$, for $\ell = 1$ to 3. The dimensions of the respective coefficient matrices are: $A(t)$ is $m \times m$, $B(t)$ is $m \times n$, $\mathbf{C}(t)$ is $m \times 1$, $G(t)$ is $m \times r$, while the $H_\ell(t)$ are dimensioned, so that $[H_1(t) \cdot \mathbf{x}] = [\sum_k H_{1ijk}(t)x_k]_{m\times q_1}$, $[H_2(t) \cdot \mathbf{u}] = [\sum_k H_{2ijk}(t)u_k]_{m\times q_2}$ and $H_3(t) = [H_{3ij}(t)]_{m\times q_3}$. The quadratic performance index or cost functional that is employed is quadratic with respect to the state and control costs, is given by the *time-to-go* or *cost-to-go* functional form:

$$V[\mathbf{X},\mathbf{U},t] = \frac{1}{2}(\mathbf{X}^\top S\mathbf{X})(t_f) + \int_t^{t_f} C(\mathbf{X}(\tau),\mathbf{U}(\tau),\tau)d\tau, \quad (2)$$

where $C(\mathbf{x},\mathbf{u},t) = \frac{1}{2}\left[\mathbf{x}^\top Q(t)\mathbf{x}, + \mathbf{u}^\top R(t)\mathbf{u}\right]$, with the time horizon (t, t_f). $S(t_f) \equiv S_f$ is the quadratic final cost coefficient matrix and $C(\mathbf{x},\mathbf{u},t)$ is quadratic instantaneous cost function. The final cost, known as the *salvage cost*, is given by the quadratic form, $\mathbf{x}^\top S_f \mathbf{x} = S_f : \mathbf{x}\mathbf{x}^\top = \mathrm{Trace}[S_f \mathbf{x}\mathbf{x}^\top]$. The coefficients $R(t)$ and $Q(t)$ are assumed to be symmetric for simplicity. The LQGP problem is defined by (1, 2).

Consider a MMS that produces the single consumable commodity. The MMS consists of k stages that form a linear sequence that is used to assemble the finished product. State dependent Poisson noises are used to model catastrophic events that affect the delivery of raw materials to the MMS. This advances the model used in [11], additionally the state equation for the number of active workstations is modeled using state dependent Poisson noise. At time t in the manufacturing planning horizon for stage i, there are $n_i(t)$ operational workstations. For each stage i, all workstations are assumed to be identical and produce goods at the same rate $c_i(t)$ with a capacity of producing M_i parts per unit time. Each workstation is subject to failure and can be repaired. The mean time between failures and the repair duration is exponentially distributed. The production rate, $c_i(t)$ is a utilization, that is the fraction of time busy. The physically realizable production rate is bounded by $0 \le c_i(t) \le c_i^{max}(t)$, where $c_i^{max}(t) = \left\{ \begin{array}{ll} s_i(t), & i = 1 \\ \min[1, \frac{s_{i-1}(t)c_{i-1}(t)n_{i-1}(t)M_{i-1}}{n_i(t)M_i}], & 1 < i \le k \end{array} \right\}$, where the maximum production rate, $c_i^{max}(t)$, is the minimum value of the physical production rate, 1.00 or full utilization, and production limitations that arise due to a shortfall of production from the previous stage due to either machine failure, strikes, or natural disasters, where $s_i(t)$ is the impact of strikes and natural disasters on stage i. The strike influence is used to limit the amount of pieces that can be produced by a given stage and is bounded by $0 \le s_i(t) \le 1$, such that $s_i(t) = 0$ means that no production can occur. In this formulation the production rate is a parameter of the dynamic system and is adjusted by the control decision.

The total number of workstations for stage i is N_i. Therefore the evolution of the number of active workstations is bounded by $0 \le n_i(t) \le N_i$, for all time, and can be viewed as a birth (repair) and death (failure) process or a random walk on the discrete interval $[0, N_i]$. The defining equation for the number of operational workstations evolves by the stochastic process using *state dependent Poisson noises* given by

$$dn_i(t) = dP^R(n_i(t), t) - dP^F(n_i(t), t), \tag{3}$$

where $dP^R(n(t), t)$ and $dP^F(n(t), t)$ are used to model the repair and failure processes, respectively, which depends on the current number of active workstations. The number of active workstations, $n_i(t)$, determines the arrival rates and mean mark amplitudes for failure and repair events respectively given by

$$1/\lambda_i^F(n_i(t), t) = \left\{ \begin{array}{ll} 0, & n_i(t) = 0 \\ 1/\lambda_i^F, & 1 \le n_i(t) \le N_i \end{array} \right\},$$
$$\overline{Z}_i^F(n_i(t), t) = \left\{ \begin{array}{ll} 0, & n_i(t) = 0 \\ \sum_{j=1}^{n_i(t)} j\Pr[n_i(t) = j], & 1 \le n_i(t) \le N_i \end{array} \right\},$$
$$1/\lambda_i^R(n_i(t), t) = \left\{ \begin{array}{ll} 1/\lambda_i^R, & 0 \le n_i(t) < N_i \\ 0, & n_i(t) = N_i \end{array} \right\},$$
$$\overline{Z}_i^R(n_i(t), t) = \left\{ \begin{array}{ll} \sum_{j=1}^{N_i - n_i(t)} j\Pr[n_i(t) = N_i - j], & 0 \le n_i(t) < N_i \\ 0, & n_i(t) = N_i \end{array} \right\}.$$

The impact of strikes and natural disasters on stage i, $s_i(t)$, evolves according to the purely stochastic equation,

$$\begin{aligned} ds_i(t) &= -d\mathbf{P}^{PS}(s_i(t), t) + d\mathbf{P}^{PR}(s_i(t), t) \\ &- d\mathbf{P}_i^{SS}(s_i(t), t) + d\mathbf{P}_i^{SR}(s_i(t), t). \end{aligned} \tag{4}$$

The term $-d\mathbf{P}^{PS}(s_i(t), t)$ is used to model the effects of primary strikes. The arrival rate for this type of strike is deterministic in the sense that normally there is a fixed date, say t_s, for the termination of a labor agreement, which if not resolved can lead to a primary strike. If a primary strike occurs at any stage, then all stages of the MMS will become idle, that is $s_i(t) = 0$ for all i. The term $d\mathbf{P}^{PR}(s_i(t), t)$ is used to model the resolution of the primary strikes which returns the state of the MMS to prestrike conditions. The term $-d\mathbf{P}_i^{SS}(s_i(t), t)$ is used to model the effects of secondary strikes and natural disasters. The arrival rate is the mean time between the occurrences of such events, while the term $d\mathbf{P}_i^{SR}(s_i(t), t)$ is used to model the resolution of secondary strikes and natural disasters.

The surplus aggregate level represents the surplus (if positive) or shortfall (if negative) of the production of pieces that have successfully completed i stages of the manufacturing process. The state equation for the surplus aggregate level for stage $i = 1$ to k is given by

$$\begin{aligned} da_i(t) &= [M_i c_i(t) n_i(t) + u_i(t) - d_i(t)]\, dt + g_i(t) dW_i(t) \\ &- I^{PS} d\mathbf{P}^{PS}(s_i(t), t) - I_i^{SS} d\mathbf{P}_i^{SS}(s_i(t), t). \end{aligned} \tag{5}$$

The change in the surplus aggregate level, $da_i(t)$, is determined by the number of pieces that have successfully completed i stages of the manufacturing process ($M_i n_i(t) c_i(t) dt$), that are not defective, and are not consumed by stage $i+1$ ($d_i(t)dt$), and by the status of the workstations. The first term, $M_i n_i(t) c_i(t) dt$, on the right hand side of (5) represents the quantity produced. The term $u_i(t)dt$ is used to adjust the production rate where the control $u_i(t)$ is expressed as the number of pieces per unit time. The term, $g_i(t)dW_i(t)$, is used to model the random fluctuations in the number of pieces produced, for example defective pieces. The demand term, $d_i(t)dt$, is the consumption of the pieces produced by stage i by stage $i+1$, $0 \le d_i(t) \le M_i N_i$. The last two terms use Poisson processes to represent the effects of strikes and natural disasters where the coefficients, I^{PS} and I_i^{SS}, are the expected value for the shortfall in the number of pieces produced. The surplus aggregate level, $a_i(t)$, for stage i is dependent on the number of operational workstations, $n_i(t)$. The birth and death process for the number of operational workstations is an *embedded Markov chain* for the surplus aggregate level. Hence, the surplus aggregate level is a piecewise continuous process whose discontinuous jumps are determined by the stochastic process for the number of operational workstations.

The cost function used is the standard *time-to-go* or *cost-to-go* form (2) that is motivated by a *zero inventory* or *Just in Time* manufacturing discipline (see Hall [4] and Bielecki and Kumar [2]) while utilizing minimum control effort. In this formulation, the salvage cost, $S(t_f)$, is used to impose a penalty on surplus or shortfall of production at the end of the planning horizon. The term $Q(t)$ is used to penalize shortfall and surplus production during the planning horizon, this term is used to maintain a strict regimen on when the consumable goods are to be produced. The term $R(t)$ is used to enforce a minimum control effort penalty.

The regular controlled production level which assumes the regular or unconstrained control, $c_i^{\text{reg}}(t) = c_i(t) + u_i^{\text{reg}}(t)/(M_i n_i(t))$ when $n_i(t) > 0$, which anticipates for the stochastic effects of workstation repair and failure, defective parts, strikes or natural disasters, and is zero when $n_i(t) = 0$. Note, that with the assumption of regular control, the surplus aggregate level will always be forced to be zero, therefore the regular controlled production level may not be physically realizable. In the case of a primary strike, $c_i(t) = 0$ for all i, the regular controlled production level which is the same as the regular control would be the number of pieces that needs to be produced to force the surplus aggregate level to zero, which clearly is not physically realizable. The constrained controlled production level, $c_i^*(t) = \min[c_i^{\text{reg}}(t), c_i^{max}(t)]$, is the restriction of the regular controlled production level to be physically realizable. The constrained controlled production rate is used as the production rate for the workstations in the state equation for the surplus aggregate level (5).

3. Numerical Example of LQGP MMS

For numerical concreteness, consider a MMS with $k = 3$ stages with a planning horizon of 100 days. Let the initial surplus aggregate level for all stages be zero, the demand be $d_i(t) = 530$ pieces per day for all stages, the total number of workstations, N_i, for each stage be 3, 5, and 4, respectively, the Gaussian random fluctuations of production is assumed absent ($g_i(t) = 0$ for $i = 1$ to 3), and that secondary strikes and natural disasters are not considered ($d\mathbf{P}_i^{SS}(s_i(t), t) = d\mathbf{P}_i^{SR}(s_i(t), t) = 0$ for $i = 1$ to

3). A single primary strike can occur at the beginning of day 63 of the planning horizon with an expected time of 14 days to resolve itself, that is the arrival rates for the strike are given by $1/\lambda^{PS}(s_i(t),t) = \left\{ \begin{array}{ll} (63-t) \text{ days}, & t < 63 \\ 0 \text{ days}, & t \geq 63 \end{array} \right\}$, and $1/\lambda^{PR}(s_i(t),t) = \left\{ \begin{array}{ll} 0 \text{ days}, & s_i(t) = 1 \\ 14 \text{ days}, & s_i(t) < 1 \end{array} \right\}$ with an impact of a shortfall of a $530 * 14 = 7420$ pieces. Note that the effects of a primary strike on the MMS disable or enable production for all stages. The operational characteristics for the workstations are summarized in the table below.

Stage i	Production Capacity, M_i (pieces/day)	Mean Time between Failure $1/\lambda_i^F$ (days)	Mean Time to Repair $1/\lambda_i^R$ (days)
1	238	85.0	0.50
2	143	75.0	0.50
3	178	90.0	0.75

Let $\Phi^R_{k,i,j}$ and $\Phi^F_{k,i,j}$ denote the discrete mark transition probabilities for the repair and failure, respectively, of $j-1$ workstations for stage k when there are i operational workstations, with transition matrices given by

$$\Phi_1^R = \begin{bmatrix} 0.00 & 0.95 & 0.05 \\ 0.00 & 1.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \end{bmatrix}, \Phi_1^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.90 & 0.10 \end{bmatrix},$$

$$\Phi_2^R = \begin{bmatrix} 0.00 & 0.90 & 0.07 & 0.02 & 0.01 \\ 0.00 & 0.92 & 0.07 & 0.01 & 0.00 \\ 0.00 & 0.93 & 0.07 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix},$$

$$\Phi_2^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.95 & 0.05 & 0.00 & 0.00 \\ 0.00 & 0.94 & 0.05 & 0.01 & 0.00 \\ 0.00 & 0.92 & 0.05 & 0.02 & 0.01 \end{bmatrix},$$

$$\Phi_3^R = \begin{bmatrix} 0.00 & 0.96 & 0.03 & 0.01 \\ 0.00 & 0.97 & 0.03 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$\Phi_3^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.95 & 0.05 & 0.00 \\ 0.00 & 0.90 & 0.07 & 0.03 \end{bmatrix},$$

The cost functional used is (2) where the coefficient matrices are given by

$$S(t_f) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & S_f & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix},$$

$$S_f = \begin{bmatrix} 11000 & 0.0 & 0.0 \\ 0.0 & 18000 & 0.0 \\ 0.0 & 0.0 & 26000 \end{bmatrix},$$

$$Q(t) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & Q_2 & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 11000 & 0.0 & 0.0 \\ 0.0 & 18000 & 0.0 \\ 0.0 & 0.0 & 25000 \end{bmatrix},$$

$$R(t) = \begin{bmatrix} 22 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 22 \end{bmatrix}.$$

By comparing the coefficients of (1) with the state equations for the MMS (3,5,4) the deterministic coefficients are given by

$$A(t) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ \text{diag}[\mathbf{M}]\text{diag}[\mathbf{c}(t)] & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix},$$

$$B(t) = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \\ 0_{3\times3} \end{bmatrix}, \qquad C(t) = \begin{bmatrix} \mathbf{0}_{3\times1} \\ -\mathbf{d}(t) \\ \mathbf{0}_{3\times1} \end{bmatrix},$$

where $\text{diag}[\mathbf{M}] = [M_i \delta_{i,j}]_{k\times k}$ is the diagonal matrix representation of the vector $\mathbf{M}$ and with the only *nonzero* stochastic process and corresponding coefficient matrix given by $d\mathbf{P}_3(\mathbf{X}(t),t) = \left[d\mathbf{P}^R(\mathbf{n}(t),t), d\mathbf{P}^F(\mathbf{n}(t),t), dP^{PS}(\mathbf{s}(t),t), dP^{PR}(\mathbf{s}(t),t)\right]^\top$,

$$H_3(t) = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7420 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7420 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7420 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Using the above numerical values and assuming the regular control the regular and constrained controlled production rates can be determined. The results are shown in Figure 1, Figure 2, and Figure 3 for stages 1, 2, and 3, respectively. At the final time of the planning horizon the percent relative error of production from the production goal is given by $[0.0844\%, 0.0279\%, -0.0401\%]^\top$, which means that the production goal has been essentially met and exceeded. The discrepancy in the percent relative error arises from not compensating for the effects of the saturation of the constrained controlled production rates due to workstation failure. This means that there are pieces that are surplus for stages 1 and 2, which the plant manager needs to consume on the remain stages in a way consistent with the production goals during the remaining time of the manufacturing horizon. When this is done, the percent relative error for all stages will be 0.0844% for all stages.

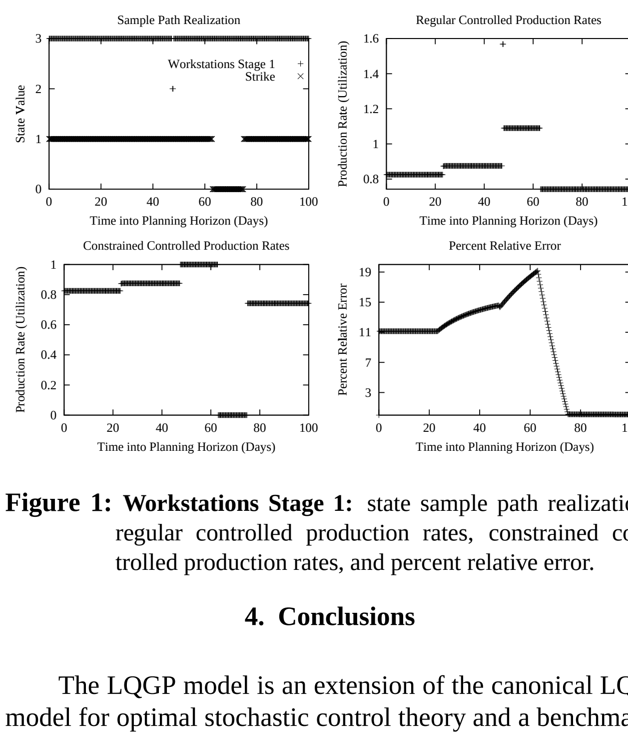

Figure 1: Workstations Stage 1: state sample path realization, regular controlled production rates, constrained controlled production rates, and percent relative error.

4. Conclusions

The LQGP model is an extension of the canonical LQG model for optimal stochastic control theory and a benchmark model for computational stochastic control for hybrid systems. The somewhat general form of the Poisson terms leads to nonlinear extensions for the usual LQG Riccati equation simplification. However, the Poisson terms and the subse-

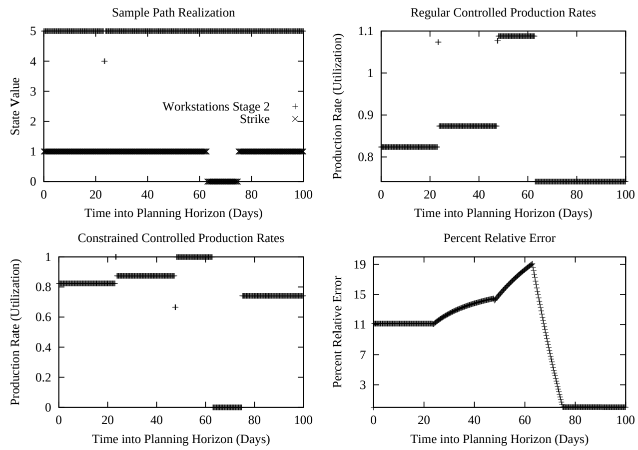

Figure 2: **Workstations Stage 2:** state sample path realization, regular controlled production rates, constrained controlled production rates, and percent relative error.

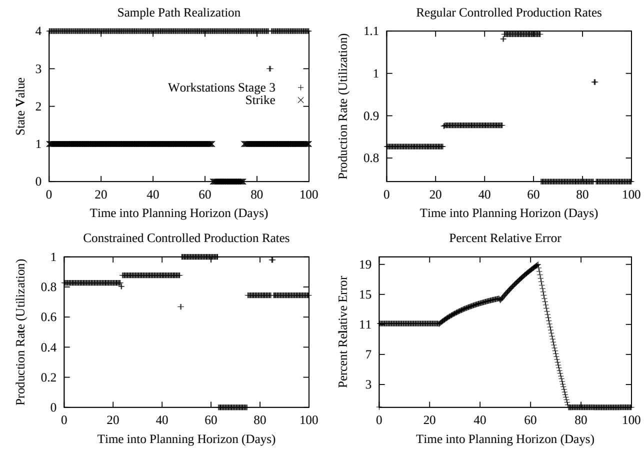

Figure 3: **Workstations Stage 3:** state sample path realization, regular controlled production rates, constrained controlled production rates, and percent relative error.

quent results are more interesting for more realistic applications, which involve discrete random jumps in continuous time, but at the cost of additional computational complexity. A sudden labor strike or natural disaster can have catastrophic consequences that are much more serious than portrayed by the typical continuous state model, in addition to the jumps due to the random failure and repair of multistage manufacturing system (MMS) workstations. Our computational procedures lead to systematic approximations to the MMS model formulated here for strikes and other random catastrophic events.

Acknowledgments

The authors would like to thank the National Center for Data Mining of the University of Illinois at Chicago for computational support through the National Scalable Cluster Project permitting use of computer facilities at the University of Pennsylvania during the development of this paper.

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