Manufacturing Production Scheduling with Preventive Maintenance in Random Environments *

J. J. Westman
Department of Mathematics
University of California
Box 951555
Los Angeles, CA 90095-1555, USA
e-mail: jwestman@math.ucla.edu
URL: http://www.math.ucla.edu/~jwestman/

and

F. B. Hanson
Laboratory for Advanced Computing
University of Illinois at Chicago
851 Morgan St.; M/C 249
Chicago, IL 60607-7045, USA
e-mail: hanson@math.uic.edu
URL: http://www.math.uic.edu/~hanson/

January 29, 2000

CCA 2000

Abstract

Consider the optimal control of a manufacturing system consisting of k stages in which a single consumable good is produced in a random jump environment. At each stage of the manufacturing process there are n workstations that can fail and be repaired. The workstations are assumed to have different operating parameters for a given stage. The mean time to failure for a given workstation, on a given stage, is modeled as a function of the uptime of the workstation. The uptime of the workstations is a monotone increasing function, which can be reset to a lower level by preventive maintenance. This formulation combines features of flexible and multistage manufacturing systems. The goal is to schedule the production of the consumable good subject to random effects and preventive maintenance.

1. Introduction

A flexible manufacturing system (FMS) is a collection of workstations that produce a family of related parts that require similar operations. A key feature of a FMS is the way in which raw materials are routed into and finished pieces are routed out of the FMS. In models for FMS, a focus is given on how a given piece travels through the system. This local perspective of how the pieces move is not included in the multistage manufacturing system (MMS) model. In a MMS the focus is on the overall throughput of the manufacturing system. Each stage of a MMS may be viewed as a FMS. The flow of pieces is modeled as a continuum and the discrete model of the FMS becomes a fluid like model. Kimemia and Gershwin [13] describe the differences and similarities between FMS and MMS, while presenting a hierarchical scheme for modeling and providing an algorithm for the operational control of a FMS. A survey of many types of real flexible manufacturing systems is given by Dupont-Gatelmand [8]. Westman and Hanson provide LQGP (Linear Quadratic Gaussian Poisson) and nonlinear models for the production scheduling of a MMS subject to workstation failure and repair [16, 17, 20] as well as strikes and natural disasters [21, 12].

For a FMS, Olsder and Suri [14], first proposed a stochastic model utilizing a homogeneous *jump Markov processes* to describe the evolution of the state of the operational (or failed) workstations. They state that the usefulness of the model is limited by the ability to solve the Hamilton-Jacobi-Bellman (HJB) partial differential equation of dynamic

*Work supported in part by the National Science Foundation Grants DMS-96-26692 and DMS-99-73231.

programming. This is due to the exponential growth of computational and memory resources needed to solve the HJB equation utilizing finite differences, commonly referred to as the *Curse of Dimensionality* [2].

Boukas and Haurie [4] present a model for a continuous-time stochastic flow control for production scheduling with preventive maintenance of the workstations. In this formulation, the mean time between failures for a given workstation is dependent on the operational age of the machine, which is defined as the time since the last restart (repair or preventive maintenance). The transition between the states of a given workstation (operational, failed, preventive maintenance) forms an irreducible Markov chain. For N workstations, there are a total of 3^N states in the Markov chain for the description of the operational state of the FMS. The variables for the system are the vector for the cumulative production surplus and the operational ages for all workstations. A numerical method based on Kushner's method is used to approximate the solution for the dynamic programming problem, which requires a discrete mesh. In this method, the mesh selected is finite and artificial boundary conditions are used which introduces additional error. In the example presented for 2 workstations and 1 part type, using 21 points for the mesh of the production surplus and operational ages, the total number of mesh points is given by $(21 * 21 * 21) * 3^2 = 83,349$. The addition of another workstation would yield a need for $(21 * 21 * 21 * 21) * 3^3 = 5,250,987$ mesh points, clearly the *Curse of Dimensionality* is present in this method, and therefore would not be suitable for a large number of workstations.

In this paper, a hybrid model of production scheduling for a manufacturing system is considered that is subject to random disturbances. The model incorporates local features of a FMS with the global perspective of a MMS. The goal of the model is generate the optimal production rates to achieve the desired production demand or profile while compensating for workstation repair, failure, and preventive maintenance. The local aspects of a FMS are utilized to describe the state of the operational workstations in the manufacturing system subject to repair, failure, and preventive maintenance. This allows for a greater realism, since each workstation can have different characteristics, as opposed to homogeneous assumptions made in many other treatments (see for example, [16, 17, 20, 21, 12]). The global aspects of a MMS are used to model the throughput of production for the manufacturing system.

In the formulation for the manufacturing system presented in this paper, the curse of dimensionality is not present. The problem formulation used is a LQGP problem (see [16]) utilizing state dependent Poisson processes (see [19]) to model the failure, repair, and preventive maintenance for the workstations. The numerical method for the LQGP problem requires the solution to a coupled set of nonlinear ordinary differential equations in *time only* and thus does not suffer from the dimensionality problems associated with partial differential equations, such as the HJB equation. The solution for the dynamic programming problem requires the multiplication of the state value by these evolutionary coefficients, and therefore no discrete mesh is necessary. The amount of memory required to solve the system is approximately V^2 where V is the total number of state variables. The operational state for all of the workstations is converted into appropriate parameters for the state dependent Poisson processes.

The paper is arranged as follows. In Section 2., a summary of the LQGP Problem with state dependent Poisson noise is given. In Section 3., a LQGP problem [16] utilizing state dependent Poisson processes [19] is used to formulate the dynamical system for the manufacturing system and in Section 4., a numerical example is presented.

2. LQGP Problem Formulation

For completeness we review in part the canonical form for the LQGP problem that originally appears in Westman and Hanson [16], for the case with state independent Poisson noise, and [19] for state dependent Poisson noise. Additionally, considerations for modeling a physical system are presented as well, as well as formal solution to the LQGP problem.

The linear-like dynamical system for the LQGP problem is governed by the stochastic differential equation (SDE) subject to Gaussian and state dependent Poisson noise disturbances is given by

$$
\begin{aligned}
d\mathbf{X}(t) &= [A(t)\mathbf{X}(t) + B(t)\mathbf{U}(t) + \mathbf{C}(t)]dt + G(t)d\mathbf{W}(t) + [H_1(t)\cdot\mathbf{X}(t)]d\mathbf{P}_1(\mathbf{X}(t),t) \\
&+ [H_2(t)\cdot\mathbf{U}(t)]d\mathbf{P}_2(\mathbf{X}(t),t) + H_3(t)d\mathbf{P}_3(\mathbf{X}(t),t), \qquad (1)
\end{aligned}
$$

for general Markov processes in continuous time, with $m \times 1$ state vector **X**(t), $n \times 1$ control vector **U**(t), $r \times 1$ Gaussian noise vector d**W**(t), and $q_\ell \times 1$ space-time Poisson noise vectors $d\mathbf{P}_\ell(\mathbf{X}(t),t)$, for $\ell = 1$ to 3. Note that the term $[H_1(t)\cdot\mathbf{X}(t)]d\mathbf{P}_1(\mathbf{X}(t),t)$ is not linear in the state. The dimensions of the respective coefficient matrices are: $A(t)$ is $m \times m$, $B(t)$ is $m \times n$, $\mathbf{C}(t)$ is $m \times 1$, $G(t)$ is $m \times r$, while the $H_\ell(t)$ are dimensioned, so that $[H_1(t)\cdot\mathbf{x}] = [\sum_k H_{1ijk}(t)x_k]_{m\times q_1}$, $[H_2(t)\cdot\mathbf{u}] = [\sum_k H_{2ijk}(t)u_k]_{m\times q_2}$ and $H_3(t) = [H_{3ij}(t)]_{m\times q_3}$. Note that the space-time Poisson terms are formulated to maintain the linear nature of the dynamics, but the first two are actually bilinear in either $\mathbf{X}$ or $\mathbf{U}$ and $d\mathbf{P}_\ell$ for $\ell = 1$ or 2, respectively.

The *state dependent Poisson noise* can be viewed as a sequence of events that is represented by its ith couple $\{T_i(\mathbf{X}(T_i)), M_i(\mathbf{X}(T_i))\}$, for $i = 1$ to k, where $T_i(\mathbf{X}(T_i))$ is the time for the occurrence of the ith jump with state dependent mark amplitude $M_i(\mathbf{X}(T_i)$. This representation of the Poisson process provides more realism and flexibility for a wider range of stochastic control applications since the arrival times and amplitudes may depend of the state of the system. Additionally, this formulation allows for simpler dynamical system modeling of complex random phenomena, but the inclusion of state dependence in the Poisson noise means that the problem is not strictly a LQGP problem in the dynamics and so it is assumed that this state dependence is not dominant.

The state dependent vector valued marked Poisson noises are related to the Poisson random measure (see Gihman and Skorohod [9] or Hanson [11]) and are defined as

$$
d\mathbf{P}_\ell(\mathbf{X}(t),t) = [dP_{\ell,i}(\mathbf{X}(t),t)]_{q_\ell\times 1} = \left[\int_{\mathcal{Z}_{\ell,i}} z\mathcal{P}_{\ell,i}(dz,\mathbf{X}(t),dt)\right]_{q_\ell\times 1}, \qquad (2)
$$

for $\ell = 1$ to 3 which consists of q_ℓ independent differentials of space-time Poisson processes that are functions of the state, $\mathbf{X}(t)$, where z is the Poisson jump amplitude random variable or the mark of the $dP_{\ell,i}(\mathbf{X}(t),t)$ Poisson process where $\ell = 1$ to 3 and $i = 1$ to q_ℓ. The mean or expectation is given by

$$
\mathrm{Mean}[d\mathbf{P}_\ell(\mathbf{X}(t),t)] = \Lambda_\ell(\mathbf{X}(t),t)dt\int_{\mathcal{Z}_\ell}\mathbf{z}\phi_\ell(\mathbf{z},\mathbf{X}(t),t)d\mathbf{z} \equiv \Lambda_\ell(\mathbf{X}(t),t)\overline{\mathbf{Z}}_\ell(\mathbf{X}(t),t)dt, \qquad (3)
$$

where $\Lambda_\ell(\mathbf{X}(t),t)$ is the diagonal matrix representation of the state dependent Poisson rates $\lambda_{\ell,i}(\mathbf{X}(t),t)$ for $\ell = 1$ to 3 and $i = 1$ to q_ℓ, $\overline{\mathbf{Z}}_\ell(\mathbf{X}(t),t)$ is the mean of the jump amplitude mark vector and $\phi_{\ell,i}(z,\mathbf{X}(t),t)$ is the density of the (ℓ,i)th amplitude mark component. Assuming component-wise independence, $d\mathbf{P}_\ell(\mathbf{X}(t),t)$ has covariance given by

$$\text{Covar}[d\mathbf{P}_\ell(\mathbf{X}(t),t), d\mathbf{P}_\ell^\top(\mathbf{X}(t),t)] = \Lambda_\ell(*)dt \int_{\mathcal{Z}_\ell} (\mathbf{z} - \overline{\mathbf{Z}}_\ell(*))(\mathbf{z} - \overline{\mathbf{Z}}_\ell(*))^\top \phi_\ell(\mathbf{z},*)d\mathbf{z} \equiv \Lambda_\ell(*)\sigma_\ell(*)dt, \tag{4}$$

with, for instance, $\sigma_\ell(*) = \sigma_\ell(\mathbf{X}(t),t) = [\sigma_{\ell,i,j}\delta_{i,j}]_{q_\ell \times q_\ell}$ denoting the diagonalized covariance of the amplitude mark distribution for $d\mathbf{P}_\ell(\mathbf{X}(t),t)$.

The Gaussian white noise term, $d\mathbf{W}(t)$, consists of r independent, standard Wiener processes $dW_i(t)$, for $i = 1$ to r. These Gaussian components have zero infinitesimal mean, Mean$[d\mathbf{W}(t)] = \mathbf{0}_{r\times 1}$ and and diagonal covariance. Covar$[d\mathbf{W}(t), d\mathbf{W}^T(t)] = I_r dt$. It is further assumed that all of the individual component terms of the Gaussian noise are independent of all of the Poisson processes.

The quadratic performance index or cost functional that is employed is quadratic with respect to the state and control costs, is given by the *time-to-go* or *cost-to-go* functional form:

$$V[\mathbf{X},\mathbf{U},t] = \frac{1}{2}(\mathbf{X}^\top S\mathbf{X})(t_f) + \int_t^{t_f} C(\mathbf{X}(\tau),\mathbf{U}(\tau),\tau)d\tau, \quad C(\mathbf{x},\mathbf{u},t) = \frac{1}{2}\left[\mathbf{x}^\top Q(t)\mathbf{x}, + \mathbf{u}^\top R(t)\mathbf{u}\right], \tag{5}$$

where the time horizon is (t, t_f), with $S(t_f) \equiv S_f$ is the quadratic final cost coefficient matrix and $C(\mathbf{x},\mathbf{u},t)$ is quadratic running cost function. In order to minimize (5) requires that the quadratic control cost coefficient $R(t)$ is assumed to be a symmetric positive definite $n \times n$ array, while the quadratic state control coefficient $Q(t)$ is assumed to be a symmetric positive semi-definite $m \times m$ array. The LQGP problem is defined by (1, 5).

For the stochastic dynamic programming formulation, the *optimal, expected cost*, is taken to be

$$v(\mathbf{x},t) \equiv \underset{\mathbf{u}[t,t_f)}{\text{Min}} \left[\underset{\mathbf{P},\mathbf{W}[t,t_f)}{\text{Mean}} [V[\mathbf{X},\mathbf{U},t] \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \right], \tag{6}$$

where the restrictions on the state and control are that they belong to the admissible classes for the state, $\mathcal{D}_\mathbf{x}$, and control, $\mathcal{D}_\mathbf{u}$, respectively. A final condition on the optimal, expected value, $v(\mathbf{x},t_f) = \frac{1}{2}\mathbf{x}^\top S_f \mathbf{x}, \;\; \text{for} \;\; \mathbf{x} \in \mathcal{D}_\mathbf{x}$, is determined from the final cost using (6) with $V[\mathbf{X},\mathbf{U},t_f]$ in (5).

Upon applying the principle of optimality to the optimal, expected performance index, (6, 5) and the chain rule for Markov stochastic processes in continuous time for the LQGP problem yields

$$\begin{aligned} 0 &= \frac{\partial v}{\partial t}(\mathbf{x},t) + \underset{\mathbf{u}}{\text{Min}} \Big[(A(t)\mathbf{x} + B(t)\mathbf{u} + \mathbf{C}(t))^T \nabla_x[v](\mathbf{x},t) \\ &+ \frac{1}{2}\left(GG^T\right)(t) : \nabla_x[\nabla_x^\top[v]](\mathbf{x},t) + \frac{1}{2}\mathbf{x}^\top Q(t)\mathbf{x} + \frac{1}{2}\mathbf{u}^\top R(t)\mathbf{u} \\ &+ \sum_{k=1}^{q_1} \lambda_{1,k}(\mathbf{x},t) \int_{\mathcal{Z}_{1,k}} [v(\mathbf{x} + [H_1(t)\cdot\mathbf{x}]_k z, t) - v(\mathbf{x},t)]\, \phi_{1,k}(z,\mathbf{x},t)dz \end{aligned}$$

$$+ \quad \sum_{k=1}^{q_2} \lambda_{2,k}(\mathbf{x},t) \int_{\mathcal{Z}_{2,k}} \left[v(\mathbf{x} + [H_2(t) \cdot \mathbf{u}]_k z, t) - v(\mathbf{x},t) \right] \phi_{2,k}(z, \mathbf{x}, t), t) dz$$

$$+ \quad \sum_{k=1}^{q_3} \lambda_{3,k}(\mathbf{x},t) \int_{\mathcal{Z}_{3,k}} \left[v(\mathbf{x} + \mathbf{H}_{3,k}(t) z, t) - v(\mathbf{x},t) \right] \phi_{3,k}(z, \mathbf{x}, t) dz \Bigg], \tag{7}$$

where the double dot product is defined by $A : B = \sum_i \sum_j A_{i,j} B_{i,j} = \mathrm{Trace}[AB^\top]$. The backward partial differential equation (PDE) (7) is known as the Hamilton-Jacobi-Bellman (HJB) equation and is subject to a final condition. The argument of the minimum is the optimal control, $\mathbf{u}^*(\mathbf{x},t)$. Regular control, $\mathbf{u}_{\text{reg}}(\mathbf{x},t)$, is the optimal control prior to the application of the control constraints.

To solve (7) subject to the final condition, for the LQGP problem a modification of the formal state decomposition of the solution for the usual LQG problem (for the usual LQG, see Bryson and Ho [6] or Dorato et al. [7]) is assumed:

$$v(\mathbf{x},t) = \frac{1}{2}\mathbf{x}^T S(t)\mathbf{x} + \mathbf{D}^T(t)\mathbf{x} + E(t) + \frac{1}{2}\int_t^{t_f} \left(GG^T\right)(\tau) : S(\tau) d\tau. \tag{8}$$

The terminal condition is satisfied, provided that

$$S(t_f) = S_f, \quad \mathbf{D}(t_f) = \mathbf{0}, \quad \text{and} \quad E(t_f) = 0. \tag{9}$$

The ansatz (8) would not, in general, be true for the state dependent case, but would be applicable if the Poisson noise is locally state independent, while globally state dependent. That is, the state domain is decomposed into subdomains, $\mathcal{D}_{\mathbf{x}} = \bigcup_i \mathcal{D}_{x_i}$, where the arrival rates and moments for all the Poisson processes are constant in the region $\mathcal{D}_{x_i}$ and can be expressed as $\Lambda(\mathbf{X}(t),t) = \Lambda_i(t)$, $\overline{\mathbf{Z}}(\mathbf{X}(t),t) = \overline{\mathbf{Z}}_i(t)$ and $\sigma(\mathbf{X}(t),t) = \sigma_i(t)$ for $\mathbf{X}(t) \in \mathcal{D}_{x_i}$, for all subdomains i. If there are any explicit dependence on $\mathbf{X}(t)$ then the resulting system would then form a LQGP/U problem (for more details see Westman and Hanson [17, 18, 19, 20]).

Assuming the ansatz (8) holds the regular, unconstrained optimal control, $\mathbf{u}^* = \mathbf{u}_{\text{reg}}$, is given by

$$\mathbf{u}_{\text{reg}}(t) = -\widehat{R}^{-1}(t)\widehat{B}^T(t)\left[S(t)\mathbf{x} + \mathbf{D}(t)\right]. \tag{10}$$

Assuming regular control, the coefficients for the optimal expected performance (8) are given by

$$0_{m\times m} = \dot{S}(t) + \left[A^T S + SA + Q\right](t) + \widetilde{\Gamma}_1(t) - \left[S\widehat{B}\widehat{R}^{-1}\widehat{B}^T S\right](t), \tag{11}$$

$$\mathbf{0}_{m\times 1} = \dot{\mathbf{D}}(t) + \left[\left(A + (\Lambda_1\overline{\mathbf{Z}}_1)^T H_1^T\right)^T \mathbf{D}\right](t) + \left[S\left(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3\right) - S\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t), \tag{12}$$

$$0 = \dot{E}(t) + \left[\left(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3\right)^T \mathbf{D}\right](t) + \frac{1}{2}\left[\left(H_3^T S H_3\right) : \Lambda_3\overline{ZZ}_3 - \mathbf{D}^T\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t), \tag{13}$$

$$\Gamma_1(t) \equiv \left[\left([H_1^T]_i S[H_1]_j : \Lambda_1\overline{ZZ}_1\right)(t)\right]_{m\times m} + 2\left[\left(\Lambda_1\overline{\mathbf{Z}}_1\right)^T H_1^T S\right](t), \tag{14}$$

$$\Gamma_2(t) \equiv \left[\left([H_2^T]_i S[H_2]_j : \Lambda_2\overline{ZZ}_2\right)(t)\right]_{n\times n}, \tag{15}$$

$$\overline{ZZ}_\ell(t) \equiv \overline{\sigma}_\ell(t) + \left(\overline{\mathbf{Z}}_\ell\overline{\mathbf{Z}}_\ell^T\right)(t) = \left[\sigma_{\ell,i}\delta_{i,j} + \overline{Z}_{\ell,i}\overline{Z}_{\ell,j}\right]_{q_\ell\times q_\ell} \tag{16}$$

for $\ell = 1$ to 3 with $\widehat{R}(t) \equiv R(t) + \widetilde{\Gamma}_2(t)$, $\widehat{B}(t) \equiv B(t) + ((\Lambda_2\overline{\mathbf{Z}}_2)^T H_2^T)(t)$, and $\widetilde{\Gamma}_\ell \equiv (\Gamma_\ell + \Gamma_\ell^T)$. Since the matrix R is positive definite, R^{-1} exists and then so does $\widehat{R}^{-1}$. Note (11) appears to have Riccati-like quadratic form, but in general is highly nonlinear due to the S dependence on $\widehat{R}$ through $\Gamma_2(t)$. If $H_\ell = [H_{\ell,i,j,k}]_{m\times q_\ell\times m_\ell}$, then $H_\ell^T = [H_{\ell,j,i,k}]_{q_\ell\times m\times m_\ell}$.

Due to uni-directional coupling of these matrix differential equations, it is assumed that the nonlinear matrix differential equation (11) for $S(t)$ is solved first and the result for $S(t)$ is substituted into equation (12) for $\mathbf{D}(t)$, which is then solved, and then both results for $S(t)$ and $\mathbf{D}(t)$ are substituted into equation (13) for the state-control independent term $E(t)$. Since the quadratic form in (8) depends only on the symmetric part of $S(t)$, only a triangle part of $S(t)$ need be solved, or $n\cdot(n+1)/2$ component equations. Thus, for the whole coefficient set $\{S(t), \mathbf{D}(t), E(t)\}$, only $n\cdot(n+1)/2+n+1$ component equations need to be solve, so that for large n the count is $\mathcal{O}(n^2/2)$, asymptotically, which is the same order of effort in getting the triangular part of $S(t)$.

3. Manufacturing System LQGP Problem Formulation

Consider a manufacturing system that produces the single consumable commodity that requires a linear sequence of k stages to assemble the finished product. The planning horizon for the manufacturing system is $[0, t_f]$. The mechanisms by which the input, *loading stage*, of raw materials and the delivery of finished products, *unloading stage*, are not considered as stages in the manufacturing system. The model presented here uses features of FMS for the active number of workstations in which the moments for the repair, failure, and preventive maintenance are functions of the state of the system, in particular the operational ages for the workstations. For similar models for FMS with variations see Akella and Kumar [1] far a treatment of optimal inventory levels, as well as Boukas and co-workers [4, 5] for a treatment which includes preventive maintenance and machine age structure.

3.1. Local Workstation State Equations

Assume that there are a total of N_k workstations for stage k. The description of the state equations for the workstations is similar in nature to that of a FMS. For workstation i, let M_{ki} denote the maximum number of pieces per unit time that can be produced. The workstations for a given stage are assumed to have different operational properties, therefore the model must account for each workstation separately. All workstations will produce goods at the same rate $c_k(t)$ for a given stage k, thereby distributing the workload across all workstations. The production rate, $c_k(t)$ is a utilization, that is the fraction of time busy, which is a parameter of the system and needs to be determined. For each workstation i on stage k, four state variables are used to describe the status of the workstation, they are the operational status, $o_{ki}(t)$, which describes the failure and repair events, the preventive maintenance status, $m_{ki}(t)$, the available production capacity $r_{ki}(t)$, and the current operational age for the workstation, $a_{ki}(t)$. The state variables for workstation i on stage k is given by:

$$\mathbf{x}_{ki}(t) = \begin{bmatrix} o_{ki}(t) \\ m_{ki}(t) \\ r_{ki}(t) \\ a_{ki}(t) \end{bmatrix}. \tag{17}$$

Each workstation is subject to failure and can be repaired and preventive maintenance is utilized to reduce the number of failures. The arrival rates, mean time till an event occurs, is a function of the operational time. This implies that the failure rate increases as time goes on, which makes preventive maintenance desirable. It is assumed that the cost incurred for preventive maintenance, and the resulting loss of production, is much less than that of workstation failure. The operational and preventive maintenance statuses evolve according to a purely stochastic differential equation that use state dependent Poisson processes. The state dependent Poisson processes allow for events to occur only when they are allowable, thus there are no problems at boundaries. The status values take values on the range from $[0, 1]$, which correspond to the percentage of available production capacity (the maximum possible rate for manufacturing). In this treatment, preventive maintenance does not have to disable the production, but may just limit the throughput of the workstation. It is assumed that for any time t, that each workstation is either operational, failed, or in maintenance, that is a machine may not be listed in more than one category. The available production capacity reflects the changes in the status variables and is used to determine the overall piece count that can be produced.

The mean time between failures and the repair duration is exponentially distributed and is a function of the current operational age. The defining equation for the operational status of workstation i on stage k is given by:

$$do_{ki}(t) = dP^R_{ki}(\mathbf{x}_{ki}(t), t) - dP^F_{ki}(\mathbf{x}_{ki}(t), t), \tag{18}$$

where $dP^F_{ki}(\mathbf{x}_{ki}(t), t)$ is used to model the failure (F) process for the workstation and $dP^R_{ki}(\mathbf{x}_{ki}(t), t)$ is used to model the repair (R) process. Preventive maintenance is performed on deterministic schedule that is based on the operational age of the workstation. The defining equation for the maintenance status is given by:

$$dm_{ki}(t) = dP^{D}_{ki}(\mathbf{x}_{ki}(t), t) - dP^{M}_{ki}(\mathbf{x}_{ki}(t), t), \tag{19}$$

where $dP^{M}_{ki}(\mathbf{x}_{ki}(t), t)$ is used to model when the workstation undergoes preventive maintenance and $dP^{D}_{ki}(\mathbf{x}_{ki}(t), t)$ is used to model the duration for the maintenance.

The events for workstation failure and preventive maintenance are mutually exclusive. Therefore, the available production capacity can be determined by using an indicator functional, $I_{ki}(t)$, given by:

$$I_{ki}(t) = \mathrm{Min}[o_{ki}(t), m_{ki}(t)]. \tag{20}$$

However, this functional does not fit in the LQGP problem paradigm. To remedy this, a new state variable for the available production capacity, $r_{ki}(t)$, taking values on the interval $[0, 1]$ is utilized that relies on the mutual exclusive properties for workstation failure and preventive maintenance, and is given by:

$$\begin{aligned} dr_{ki}(t) &= do_{ki}(t) + dm_{ki}(t) \\ &= dP^{R}_{ki}(\mathbf{x}_{ki}(t), t) - dP^{F}_{ki}(\mathbf{x}_{ki}(t), t) + dP^{D}_{ki}(\mathbf{x}_{ki}(t), t) - dP^{M}_{ki}(\mathbf{x}_{ki}(t), t). \end{aligned} \tag{21}$$

Therefore, at time t, workstation i of stage k has a maximum production capacity of pieces per unit time given by

$$\widehat{M}_{ki}(t) = M_{ki} r_{ki}(t), \tag{22}$$

therefore the maximum production for the stage is given by

$$\overline{M}_{k}(t) = \mathbf{M}_{k}^{\top} \mathbf{r}_{k}(t), \tag{23}$$

where

$$\mathbf{M}_{k} = \begin{bmatrix} M_{k1} \\ \cdots \\ M_{kN_k} \end{bmatrix}_{N_k \times 1}, \quad \mathbf{r}_{k}(t) = \begin{bmatrix} r_{k1}(t) \\ \cdots \\ r_{kN_k}(t) \end{bmatrix}_{N_k \times 1}. \tag{24}$$

The state dependent Poisson processes in (18), (19), and (21) consist of an arrival and amplitude processes, which depend on the current state of the workstation. The sojourn times for the failure processes, $dP^{F}_{ki}(\mathbf{x}_{ki}(t), t)$, and repair processes, $dP^{R}_{ki}(\mathbf{x}_{ki}(t), t)$, are given by

$$\frac{1}{\lambda^{F}_{ki}(\mathbf{x}_{ki}(t), t)} = \left\{ \begin{array}{ll} T^{F}_{ki} - a_{ki}(t), & r_{ki}(t) = 1 \\ 0, & \text{otherwise} \end{array} \right\}, \tag{25}$$

and

$$\frac{1}{\lambda_{ki}^{R}(\mathbf{x}_{ki}(t),t)} = \left\{ \begin{array}{ll} T_{ki}^{R}, & o_{ki}(t)=0 \\ 0, & o_{ki}(t)=1 \end{array} \right\}, \tag{26}$$

where T_{ki}^{F} and T_{ki}^{R} are the mean times between failure and repair, respectively. The amplitudes for these processes are $\overline{Z}_{ki}^{F} = \overline{Z}_{ki}^{R} = 1$. The sojourn times for preventive maintenance processes, $dP_{ki}^{M}(\mathbf{x}_{ki}(t),t)$, and the processes corresponding to the duration, $dP_{ki}^{D}(\mathbf{x}_{ki}(t),t)$, are given by

$$\frac{1}{\lambda_{ki}^{M}(\mathbf{x}_{ki}(t),t)} = \left\{ \begin{array}{ll} T_{ki}^{M} - a_{ki}(t), & r_{ki}(t)=1 \\ 0, & \text{otherwise} \end{array} \right\}, \tag{27}$$

and

$$\frac{1}{\lambda_{ki}^{D}(\mathbf{x}_{ki}(t),t)} = \left\{ \begin{array}{ll} T_{ki}^{D}, & 0 \le m_{ki}(t) < 1 \\ 0, & m_{ki}(t)=1 \end{array} \right\}, \tag{28}$$

where T_{ki}^{M} and T_{ki}^{D} are the mean times between maintenance and its duration, respectively. The amplitude for the preventive maintenance, $\overline{Z}_{ki}^{M}$, should be modeled as the mean percent of production capacity lost on the interval $[0,1]$, where the value 1 is interpreted as fully disabling the workstation. The amplitude for the duration of the maintenance should be the same as for the maintenance, $\overline{Z}_{ki}^{D} = \overline{Z}_{ki}^{M}$.

The current operational age of a workstation is a monotone increasing function of time and the number of pieces produced based on the production rate $c_k(t)$. The operational age of the workstation is reset to a lower value, for simplicity 0, upon the completion of workstation repair or maintenance is given by:

$$da_{ki}(t) = f(c_k(t),t)dt - H_{ki}^{D}(t)dP_{ki}^{D}(\mathbf{x}_{ki}(t),t) - H_{ki}^{R}(t)dP_{ki}^{R}(\mathbf{x}_{ki}(t),t), \tag{29}$$

where $H_{ki}^{D}(t)$ and $H_{ki}^{R}(t)$ are the coefficients that are used to reset (here, zero out), the operational age due to maintenance and repair, respectively, with

$$a_{ki}(\tau_{ki}) = 0 \tag{30}$$

where τ_{ki} is the time of the last reset.

3.2. Global Surplus State Equations

The goal of the global surplus state equations is track the production for each stage k of the manufacturing system to a specified demand function, $d_k(t)$, which is expressed as the number of pieces per unit time. The state variable used for this tracking problem is the surplus aggregate level, $s_i(t)$, which represents the surplus (if positive) or shortfall (if negative) of the production of pieces that have successfully completed stage i of the manufacturing process, where

$i = 1$ to k. The ideal for the manufacturing system is to have $s_i(t) = 0$ for all time t in the production horizon for every stage i. The control $u_k(t)$, expressed as the number of pieces per unit time, is used to adjust the production rates to compensate for all random effects in the manufacturing system such as workstation failure, repair, and preventive maintenance as well as small local effects modeled as a Gaussian noise for example defective pieces. In the unconstrained case, the control can be selected so that the production goal for all time is satisfied for all stages, that is $s_i(t)$. However, the resulting production rates may not be physically realizable.

The state equation for the surplus aggregate level for stage $i = 1$ to k is given by

$$ds_i(t) = \left[\mathbf{M}_i^\top \mathbf{r}_i(t) c_i(t) + u_i(t) - d_i(t)\right] dt + g_i(t) dW_i(t). \tag{31}$$

The change in the surplus aggregate level, $ds_i(t)$, is determined by the number of pieces that have successfully completed i stages of the manufacturing process ($\mathbf{M}_i^\top \mathbf{r}_i(t) c_i(t) dt$), that are not defective, and are not consumed by stage $i+1 \quad (d_i(t)dt)$. The first term, $\mathbf{M}_i^\top \mathbf{r}_i(t) c_i(t) dt$, on the right hand side of (31) represents the quantity produced which depends on the number of operational workstations for stage i. The term $u_i(t)dt$ is used to adjust the production rate. The term, $g_i(t) dW_i(t)$, is used to model the random fluctuations in the number of pieces produced, for example defective pieces. The demand or consumption term, $d_i(t)dt$, is the consumption of the pieces produced by stage i by stage $i+1$.

The surplus aggregate level, $s_i(t)$, for stage i is dependent on the number of operational workstations. The processes for the failure, repair, and preventive maintenance for the workstations is an *embedded Markov chain* (see Taylor and Karlin [15], for instance), for the surplus aggregate level. These events are used to describe the sojourn times for the discontinuous jumps in the surplus aggregate level. Hence, the surplus aggregate level is a piecewise continuous process whose discontinuous jumps are determined by the stochastic processes of the workstations.

The demand rate $d_i(t)$ is the number of parts needed per unit time to insure that the manufacturing process is a continuous flow of work, so that the desired number of completed pieces are produced. The demand rate must also take into account, based on past history, a minimal buffer level sufficient to compensate for defective pieces as well as workstation failures, and to insure that the proper start-up surplus aggregate levels are present for the next planning horizon. In order for the manufacturing system to be well posed, it is required for all time t that

$$0 \leq d_i(t) \leq M \tag{32}$$

where the minimum production (manufacturing bottleneck) is given by

$$M = \operatorname*{Min}_k \left[\sum_{i=1}^{N_k} M_{ki} \right] \tag{33}$$

so that the production goal of the manufacturing system is attainable.

3.3. Cost Functional

The cost function used is the standard *time-to-go* or *cost-to-go* form (5), that is motivated by a *zero inventory* or *Just in Time* manufacturing discipline (see Hall [10] and Bielecki and Kumar [3]) while utilizing minimum control effort. In this formulation, the salvage cost, $S(t_f)$, is used to impose a penalty on surplus or shortfall of production at the end of the planning horizon. The term $Q(t)$ is used to penalize shortfall and surplus production during the planning horizon, this term is used to maintain a strict regimen on when the consumable goods are to be produced. The term $R(t)$ is used to enforce a minimum control effort penalty.

3.4. Manufacturing Model Outputs

To solve this problem, assume the regular or unconstrained control (10) and solve the nonlinear system of ordinary differential equations (11,12,13). This allows the plant manager of the manufacturing system to calculate the desired or ideal production rate and the physically realizable production rate. With these production rates, the plant manager can project over the remaining production horizon the expected deviation from the final production goal.

Let $n_i(t)$ denote the number of operational workstations on stage i and is given by

$$n_i(t) = \mathbf{1}^\top \mathbf{r}_i(t), \tag{34}$$

where $\mathbf{1}$ is a $N_k \times 1$ vector whose elements are 1. The regular controlled production level for stage i anticipates for the stochastic effects of workstation failure, repair, and maintenance as well as defective parts is given by

$$c_i^{\text{reg}}(t) = \left\{ \begin{array}{ll} 0, & n_i(t) = 0 \\ c_i(t) + \dfrac{u_i^{\text{reg}}(t)}{\mathbf{M}_i^\top \mathbf{r}_i(t)}, & n_i(t) > 0 \end{array} \right\}, \tag{35}$$

where $u_i^{\text{reg}}(t)$ is the regular control. Note that with the assumption of regular control, the surplus aggregate level will always be forced to be zero, therefore the regular controlled production level may not be physically realizable. The constrained controlled production level, $c_i^*(t)$, is the restriction of the regular controlled production level to be physically realizable and is given by

$$c_i^*(t) = \min[c_i^{\text{reg}}(t), c_i^{max}(t)], \tag{36}$$

where $c_i^{max}(t)$ is defined as

$$c_i^{max}(t) = \left\{ \begin{array}{ll} 1, & i = 1 \\ \min[1, \dfrac{c_{i-1}^*(t)\mathbf{M}_{i-1}^\top \mathbf{r}_{i-1}(t)}{\mathbf{M}_i^\top \mathbf{r}_i(t)}], & 1 < i \leq k \end{array} \right\}, \tag{37}$$

where the maximum production rate, $c_i^{max}(t)$, is the minimum value of the physical production rate, 1.00 or full utilization, and production limitations that arise due to a shortfall of production from the previous stage due to either machine failure, maintenance, or defective pieces. The constrained controlled production rate is used to set the production rate for the workstations.

4. Numerical Example of LQGP MMS

For numerical concreteness, consider a manufacturing system with $k = 2$ stages with a planning horizon of 80 hours. Let the initial surplus aggregate level for all stages be zero, the demand be 162 pieces per hour for all stages ($d_1(t) = d_2(t) = 185$), the total number of workstations, N_i, for each stage be 3 and 2, respectively, the Gaussian random fluctuations of production is assumed absent ($g_i(t) = 0$ for $i = 1$ and 2). The operational characteristics for the workstations are summarized in the table below. During preventive maintenance and workstation failure no production occurs. Therefore, the moments for the moments for the state dependent Poisson processes in (18), (19), (21), and (29) are given by $\overline{Z}^F_{ki} = \overline{Z}^R_{ki} = \overline{Z}^D_{ki} = \overline{Z}^M_{ki} = 1$ with all covariances being 0.

Assume that when the operational age of a workstation (29) is reset either due to a repair or preventive maintenance the operational age of the workstation is set to zero and that the aging process is based on the amount of time operational only. This implies that,

$$f(c_k(t), t)dt = 1, \qquad H^D_{ki}(t) = H^R_{ki}(t) = \tau_{ki} - t - a_{ki}(\tau_{ki}), \tag{38}$$

where τ_{ki} is the time of the last reset (initial value is 0) and $a_{ki}(\tau_{ki})$ is viewed as a parameter that represents the age of the workstation at the last reset, which is zero for all $\tau_{ki} \neq 0$ and is specified in the table below for $\tau_{ki} = 0$.

Stage k	Workstation i	Production Capacity, M_{ki} (pieces/hour)	Operational Age, $a_{ki}(0)$ (hours)	Mean Times (hours) T^F_{ki}	T^R_{ki}	T^M_{ki}	T^D_{ki}
1	1	60	10	120	6	70	1
1	2	70	60	140	8	90	2
1	3	75	80	140	7	90	2
2	1	120	10	120	8	95	2
2	2	110	0	120	6	85	2

This manufacturing system consists of 20 local and 2 global state variables for a state of dimension 22. Define the local state vectors as

$$\mathbf{o}(t) = \begin{bmatrix} o_{11}(t) \\ o_{12}(t) \\ o_{13}(t) \\ o_{21}(t) \\ o_{22}(t) \end{bmatrix}, \quad \mathbf{m}(t) = \begin{bmatrix} m_{11}(t) \\ m_{12}(t) \\ m_{13}(t) \\ m_{21}(t) \\ m_{22}(t) \end{bmatrix}, \quad \mathbf{r}(t) = \begin{bmatrix} r_{11}(t) \\ r_{12}(t) \\ r_{13}(t) \\ r_{21}(t) \\ r_{22}(t) \end{bmatrix}, \quad \mathbf{a}(t) = \begin{bmatrix} a_{11}(t) \\ a_{12}(t) \\ a_{13}(t) \\ a_{21}(t) \\ a_{22}(t) \end{bmatrix} \tag{39}$$

for the operational status, preventive maintenance status, and current operational age, respectively. Define the global state vector for the surplus aggregate level as

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}. \tag{40}$$

The total state and control vectors are given by

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{o}(t) \\ \mathbf{m}(t) \\ \mathbf{r}(t) \\ \mathbf{a}(t) \\ \mathbf{s}(t) \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \tag{41}$$

The cost functional used is (5) where the coefficient matrices are given by

$$S(t_f) = \begin{bmatrix} 0_{20\times 20} & 0_{20\times 2} \\ 0_{2\times 20} & S_f \end{bmatrix}, \quad S_f = \begin{bmatrix} 11000 & 0 \\ 0 & 18000 \end{bmatrix},$$

$$Q(t) = \begin{bmatrix} 0_{20\times 20} & 0_{20\times 2} \\ 0_{2\times 20} & Q_2 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 9000 & 0 \\ 0 & 15000 \end{bmatrix}, \quad R(t) = \begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix}.$$

By comparing the coefficients of (1) with the state equations for the manufacturing system (18), (19), (21), (29), and (31) the deterministic coefficients are given by

$$A(t) = \begin{bmatrix} 0_{20\times 10} & 0_{20\times 5} & 0_{20\times 7} \\ 0_{2\times 10} & M_A(t) & 0_{2\times 7} \end{bmatrix}, \qquad B(t) = \begin{bmatrix} 0_{20\times 2} \\ I_{2\times 2} \end{bmatrix}, \qquad C(t) = \begin{bmatrix} \mathbf{0}_{15\times 1} \\ \mathbf{1}_{5\times 1} \\ d_1(t) \\ d_2(t) \end{bmatrix}, \tag{42}$$

where

$$M_A(t) = \begin{bmatrix} M_{11}c_1(t) & M_{12}c_1(t) & M_{13}c_1(t) & 0 & 0 \\ 0 & 0 & 0 & M_{21}c_2(t) & M_{22}c_2(t) \end{bmatrix}.$$

The only *nonzero* stochastic process and corresponding coefficient matrix given by

$$d\mathbf{P}_3(\mathbf{X}(t),t) = \begin{bmatrix} d\mathbf{P}^R(\mathbf{X}(t),t) \\ d\mathbf{P}^F(\mathbf{X}(t),t) \\ d\mathbf{P}^D(\mathbf{X}(t),t) \\ d\mathbf{P}^M(\mathbf{X}(t),t) \end{bmatrix}, \qquad H_3(t) = \begin{bmatrix} I_{5\times5} & -I_{5\times5} & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & I_{5\times5} & -I_{5\times5} \\ I_{5\times5} & -I_{5\times5} & I_{5\times5} & -I_{5\times5} \\ -H_3^R(t) & 0_{5\times5} & -H_3^D(t) & 0_{5\times5} \\ 0_{2\times5} & 0_{2\times5} & 0_{2\times5} & 0_{2\times5} \end{bmatrix},$$

with

$$-H_3^R(t) = -H_3^D(t) = \operatorname{diag}\begin{bmatrix} \tau_{11} - t - a_{11}(\tau_{11}) \\ \tau_{12} - t - a_{12}(\tau_{12}) \\ \tau_{13} - t - a_{13}(\tau_{13}) \\ \tau_{21} - t - a_{21}(\tau_{21}) \\ \tau_{22} - t - a_{22}(\tau_{22}) \end{bmatrix},$$

where $\operatorname{diag}[\mathbf{v}] = [v_i\delta_{i,j}]_{k\times k}$ is the diagonal matrix representation of the $k\times1$ vector $\mathbf{v}$ and the state dependent Poisson processes are given by

$$d\mathbf{P}^R(\mathbf{X}(t),t) \begin{bmatrix} d\mathbf{P}^R_{11}(\mathbf{X}(t),t) \\ d\mathbf{P}^R_{12}(\mathbf{X}(t),t) \\ d\mathbf{P}^R_{13}(\mathbf{X}(t),t) \\ d\mathbf{P}^R_{21}(\mathbf{X}(t),t) \\ d\mathbf{P}^R_{22}(\mathbf{X}(t),t) \end{bmatrix}, \qquad d\mathbf{P}^F(\mathbf{X}(t),t) \begin{bmatrix} d\mathbf{P}^F_{11}(\mathbf{X}(t),t) \\ d\mathbf{P}^F_{12}(\mathbf{X}(t),t) \\ d\mathbf{P}^F_{13}(\mathbf{X}(t),t) \\ d\mathbf{P}^F_{21}(\mathbf{X}(t),t) \\ d\mathbf{P}^F_{22}(\mathbf{X}(t),t) \end{bmatrix},$$

$$d\mathbf{P}^D(\mathbf{X}(t),t) \begin{bmatrix} d\mathbf{P}^D_{11}(\mathbf{X}(t),t) \\ d\mathbf{P}^D_{12}(\mathbf{X}(t),t) \\ d\mathbf{P}^D_{13}(\mathbf{X}(t),t) \\ d\mathbf{P}^D_{21}(\mathbf{X}(t),t) \\ d\mathbf{P}^D_{22}(\mathbf{X}(t),t) \end{bmatrix}, \qquad d\mathbf{P}^M(\mathbf{X}(t),t) \begin{bmatrix} d\mathbf{P}^M_{11}(\mathbf{X}(t),t) \\ d\mathbf{P}^M_{12}(\mathbf{X}(t),t) \\ d\mathbf{P}^M_{13}(\mathbf{X}(t),t) \\ d\mathbf{P}^M_{21}(\mathbf{X}(t),t) \\ d\mathbf{P}^M_{22}(\mathbf{X}(t),t) \end{bmatrix}.$$

Using the above numerical values and assuming the regular control the temporal dependent coefficients $S(t)$, $d\mathbf{D}(t)$, and $E(t)$ can be determined from (11,12,13). With the temporal coefficients known the regular control can be determined from (10) for any state value. Finally, the regular control and value for the state can be used to determine the regular controlled production rate, $c_i^{\mathrm{reg}}(t)$, and constrained controlled production rate, $c_i^*(t)$, and the deviation from the target production goal, $\widehat{s}_i(t,tf)$. This information can be precomputed and stored in a database that can then be used to control the production of an automated manufacturing system.

Consider the sample path trajectory described in the table below.

Event Time (hours)	Stage	Workstation	Type	Duration (hours)
15	1	3	failure	7
30	1	2	maintenance	2
60	1	1	maintenance	2

The constrained and regular controlled production rates for the manufacturing system are given in Figure 1. These production rates show the anticipation of workstation repair and failure. In Figure 2, the percent relative error is given. At the final time of the planning horizon the percent relative error is $(0.11183, -0.87636)^\top$. One drawback of the model is that it only considers feed forward effects. In this example, more pieces are produced on stage 1 then consumed on stage 2. The plant manager should adjust the production rates to consume the excess production from stage 1, which would result in a 0.11183 percent error for manufacturing system. The results presented here required 66 CPU seconds and 68 wallclock seconds to complete on a Sun Ultra 5, with a memory demand of 1.75 megabytes.

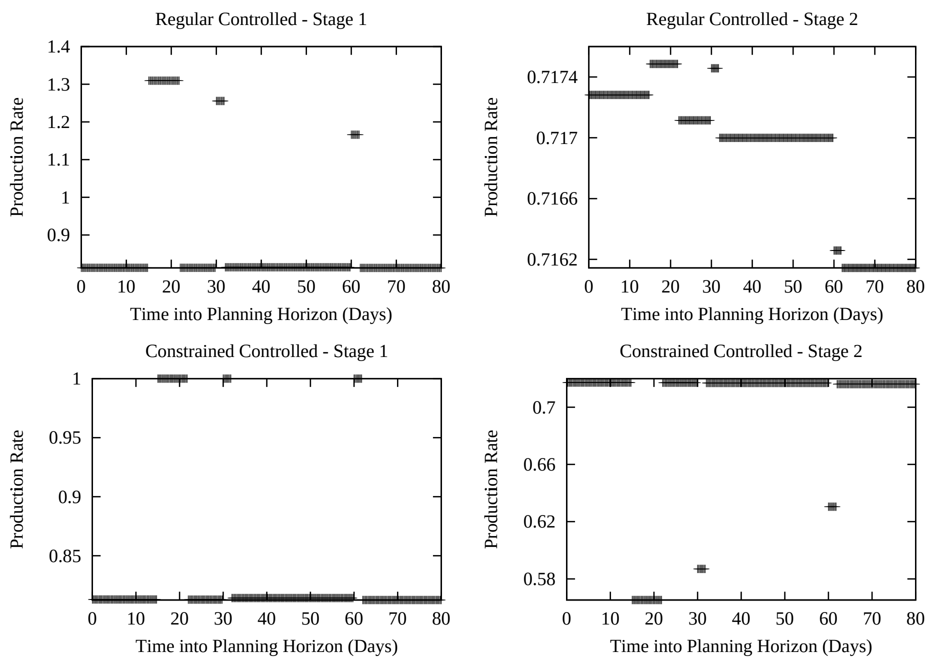

Figure 1: Regular and constrained controlled production rates for stages 1 and 2.

5. Conclusions

The LQGP model is an extension of the continuous sample path LQG model for optimal stochastic control theory and is a benchmark model for computational stochastic control for hybrid systems in which discontinuous paths are permitted. Here we have relaxed the linear dynamics assumption of LQGP by allowing the space-time Poisson noise to be state dependent. The somewhat general form of the Poisson terms leads to nonlinear extensions for the

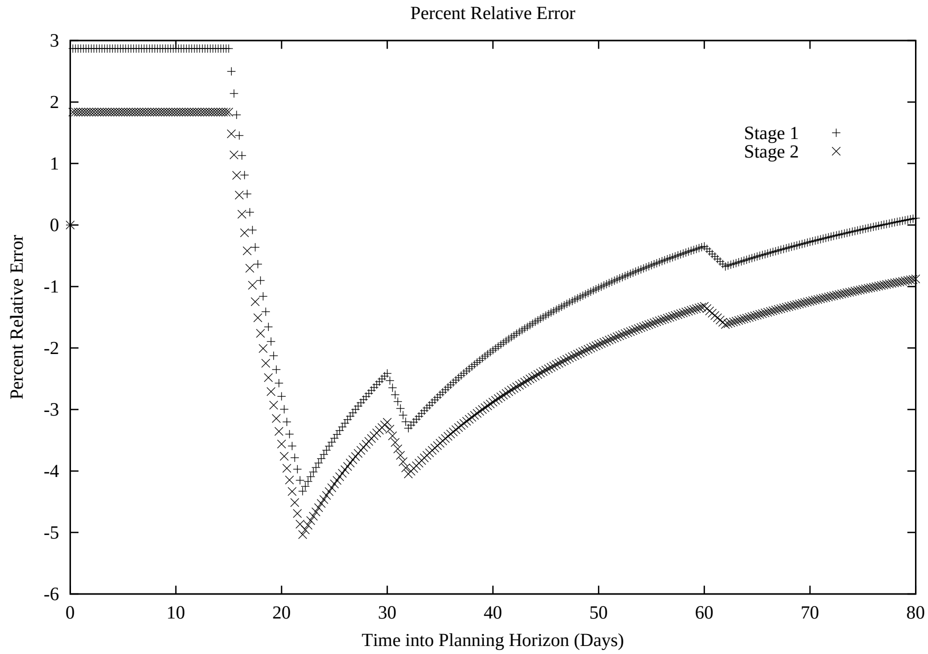

Figure 2: Percent relative error for stages 1 and 2.

standard LQG Riccati equation. However, the Poisson terms and the subsequent results are more interesting for more realistic applications, which involve discrete random jumps in the sample paths in continuous time, but at the cost of additional computational complexity. Preventive maintenance can extend the life of a workstation and thereby insure the stability of a manufacturing system. The inclusion of preventive maintenance in this model, which results in discontinuous jumps in the state value, adds more realism that is much more important than those modeled by continuous state models, and then there are the additional jumps due to the random failure and repair of manufacturing system workstations. Our computational procedures will lead to systematic approximations to the manufacturing system model formulated here for preventive maintenance and other random catastrophic events.

References

[1] R. Akella and P. R. Kumar, ”Optimal Control of Production Rate in a Failure Prone Manufacturing System,” *IEEE Trans. Autom. Control*, vol. 31, pp. 116-126, February 1986.

[2] R. E. Bellman, *Dynamic Programming*, Princeton University Press, Princeton, NJ, 1957.

[3] T. Bielecki and P. R. Kumar, ”Optimality of Zero-Inventory Policies for Unreliable Manufacturing Systems,” *Operations Research*, vol. 36, pp. 532-541, July-August 1988.

[4] E. K. Boukas and A. Haurie, ”Manufacture Flow Control and Preventive Maintenance: A Stochastic Control Approach,” *IEEE Trans. Autom. Control*, vol. 35, pp. 1024-1031, September 1990.

[5] E. K. Boukas and H. Yang, ”Optimal Control of Manufacturing Flow and Preventive Maintenance,” *IEEE Trans. Autom. Control*, vol. 41, pp. 881-885, June 1996.

[6] A. E. Bryson and Y. Ho, *Applied Optimal Control*, Ginn, Waltham, 1975.

[7] P. Dorato, C. Abdallah, and V. Cerone, *Linear-Quadratic Control: An Introduction*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

[8] C. Dupont-Gatelmand, ”A Survey of Flexible Manufacturing Systems,” *J. Manufacturing Systems*, vol. 1, pp. 1-16, 1982

[9] I. I. Gihman and A. V. Skorohod, *Stochastic Differential Equations*, Springer-Verlag, New York, 1972.

[10] R. W. Hall, *Zero Inventories*, Dow Jones-Irwin, Homewood, Illinois, 1983.

[11] F. B. Hanson, ”Techniques in Computational Stochastic Dynamic Programming,” *Digital and Control System Techniques and Applications*, edited by C. T. Leondes, Academic Press, New York, pp. 103-162, 1996.

[12] F. B. Hanson and J. J. Westman, ”Computational Stochastic Multistage Manufacturing Systems with Strikes and Other Adverse Random Events,” to appear in *Proceedings of 14th International Conference on Mathematical Theory of Networks and systems Systems (MTNS2000)*, 10 pages in ms., 12 December 1999.

[13] J. Kimemia and S. B. Gershwin, ”An Algorithm for the Computer Control of a Flexible Manufacturing System,” *IIE Trans.*, vol. 15, pp. 353-362, December 1983.

[14] G. J. Olsder and R. Suri, ”Time-Optimal Control of Parts-Routing in a Manufacturing System with Failure-Prone Machines,” *Proceedings of the 19th IEEE Conference on Decision and Control*, pp. 722-727, 1980.

[15] H. M. Taylor and S. Karlin, *An Introduction to Stochastic Modeling*, Academic Press, Inc., San Diego, 1984.

[16] J. J. Westman and F. B. Hanson, ”The LQGP Problem: A Manufacturing Application,” *Proceedings of the 1997 American Control Conference*, vol. 1, pp. 566-570, 1997.

[17] J. J. Westman and F. B. Hanson, ”The NLQGP Problem: Application to a Multistage Manufacturing System,” *Proceedings of the 1998 American Control Conference*,vol. 2, pp 1104-1108, 1998.

[18] J. J. Westman and F. B. Hanson, ”Computational Method for Nonlinear Stochastic Optimal Control,” *Proceedings of the 1999 American Control Conference*, pp 2798-2802, 1999.

[19] J. J. Westman and F. B. Hanson, ”State Dependent Jump Models in Optimal Control,” *Proceedings of the 1999 Conference on Decision and Control*, pp. 2378-2383, 1999.

[20] J. J. Westman and F. B. Hanson, ”Nonlinear State Dynamics: Computational Methods and Manufacturing Example,” to appear in *International Journal of Control*, 17 pages in galleys, 12 November 1999.

[21] J. J. Westman and F. B. Hanson, ”MMS Production Scheduling Subject to Strikes in Random Environments,” accepted in *Proceedings of 2000 American Control Conference*, 15 pages in ms., June 2000.