Ideas and Technology of Control Systems

NSF Workshop for Middle School and High School Students and Teachers
by Molly H. Shor and Floyd B. Hanson

The IEEE Control Systems Society's Technical Committee on Control Education strives to bring control system concepts and technologies to the awareness of high school and middle school students and teachers. Control is used in many common devices and systems, such as computer hard drives, VCRs, automobiles, and aircraft, but is usually hidden from view. The goal of this Committee is to promote an increased awareness among students and teachers of the importance and cross-disciplinary nature of control and systems technology.

To help meet these objectives, a workshop was held at the 42nd IEEE Conference on Decision and Control (CDC 2003) in Maui, Hawaii, on Tuesday, December 9th, 2003. About 250 students from the Maui School District, which spans the islands of Maui, Lanai and Molokai, attended this workshop, escorted by their teachers (Figure 1). Teachers who had attended earlier workshops came from as far away as Florida to participate. Because of space limitations, several hundred additional students had to be turned away from this extraordinary field trip.

Bozenna Pasik-Duncan, Chair of the Technical Committee on Control Education and CDC 2003's Chair for Control Education Activities, organized this workshop with sponsorship from CDC 2003 and funding from the National Science Foundation (NSF) and the University of Kansas. Dr. Kishan Baheti, Program Director in NSF's Division of Electrical and Communications Systems, provided the NSF funding. The Technical Committee on Control Education has organized similar workshops at a number of earlier conferences. Workshops for high school teachers were held at the American Control Conferences in Chicago (2000) and Denver (2003). A workshop for high school students was held at the 41st IEEE Conference on Decision and Control in Las Vegas, Nevada, in December 2002.

Control system experts from our technical community were recruited to present at the workshop based on their willingness and ability to present control topics at an appropriate level for secondary school students. These experts sought to describe control and system technology ideas to these students and teachers and to expose them to control applications and research. This workshop's presenters included Raffaello D'Andrea of Cornell University, Christos Cassandras of Boston University, Theodore Djaferis of the University of Massachusetts, Richard Murray of the California Institute of Technology, Mark Spong of the University of Illinois, and Katsuhisa Furuta of the Tokyo Denki University. In addition, Brian Rosen of Pixar Animation Studios, Kishan Baheti of the National Science Foundation), Shane Haas of AlphaSimplex Group, and Suzanne Lenhart of the University of Tennessee presented related dynamic system, nanoscale system, and probability topics.

The workshop began with welcomes from dignitaries from the National Science Foundation, the Control Systems Society and CDC 2003 (Figure 2). These included Vasundara Varadan, NSF Division Director for Electrical and Communications Systems, Control Systems Society President Cheryl Schrader, and CDC 2003 General Chair Frank Lewis, among others. Technical Committee Chair Bozenna Pasik-Duncan opened the workshop.

During the course of the day, the students and teachers learned about the power of feedback, the joys and perils of automation, robots playing soccer, and an autonomous

vehicle race. They also heard about how animation and lighting of characters is done for movies, the control of inverted pendula, embedded systems, mechatronics, the new field of nanotechnology, and the importance of information in the financial markets. The students played a game, developed for the high school level, that demonstrates the non-transitivity of probabilistic phenomena.

The National Science Foundation offers support to those researchers who want to involve K-12 teachers in their funded research or to assist in the development of suitable materials for K-12 classrooms. Baheti shared information about the NSF Research Experiences for Teachers program, which support the involvement of a teacher in funded research. The Technical Committee on Control Education also plans to assist in the development of appropriate materials so that students who are unable to attend a workshop such as this one can still be exposed to these topics. With the appropriate lesson materials, and the appropriate exposure themselves, teachers will be able to bring these topics into their regular classes.

The Power of Feedback

Ted Djaferis gave several examples illustrating the use of feedback, including walking, body temperature regulation, and collision avoidance in automobiles.

When walking, we use our eyes to sense our position and observe our environment. We continuously monitor our position and compare it to our desired trajectory. We adjust our steps to move closer to our desired trajectory and avoid obstacles in our way. Taking our current sensed position and using this information to plan our future steps is referred to as a feedback strategy. Consider how difficult it is for blind people to walk because of the absence of feedback. A walking stick is used to partially replace feedback capabilities.

The temperature in the human body is regulated naturally. Whether we are in the sun or in an air-conditioned room, our body temperature stays close to our normal body temperature. This normal temperature may vary from one person to another or from one time in our biological cycle to another, but it varies relatively little in response to changes in our external environment.

An example of feedback that is local to Hawaii is the outrigger canoe. The outrigger canoe made possible travel across the Pacific Ocean in ancient times. Ted previewed this example and that Danny Abramovitch would speak of this in detail in a plenary talk the next day. (See the Outrigger sidebar for more information.)

When we drive, we avoid collisions by driving at a safe distance from other cars. This means that we must monitor the distance and modify our actions depending on what we sense. Suppose we wish to drive 30 feet behind another vehicle. We must look to see if we are closer or farther than that and adjust our position accordingly. A functional diagram of how this is accomplished by the typical driver is given in Figure 3. Luxury cars such as Mercedes or Lexus models have systems that "automate" this process by employing a vehicle radar system that either warns the driver or applies partial braking.

Ted shared a photo and video of CIMCAR (Computer Intelligent Model CAR), a small model car operated by a servo motor, powered by batteries, with its speed controlled by the current to the motor (Figure 4). Without feedback, the car can be made to stop at the right place before a wall if it is on a level surface by turning on and off the motor at

the appropriate pre-computed times. However, if the car is then placed on a ramp sloping down and uses the same pre-computed on-off times, it slams into the wall. If the distance between the car and the wall is measured (by a sonar sensor) and that information is used to determine how to control the motor (feedback), the car again can be made to stop at the desired distance away from the wall. Sensors, actuators, and micro-controllers are used to implement the control strategy. The algorithms are first tested using computer simulation, before experiments are conducted on the CIMCAR.

Control engineers solve automatic control problems in service to humanity using science, mathematics, and engineering principles and practices, along with software and hardware. They apply their work to a very diverse set of systems that may have electrical, mechanical, chemical, hydraulic, financial or biological characteristics. Automatic control is a fascinating field of study. It is universal, multidisciplinary, helps one develop a systems viewpoint in the solution of problems, and allows one to work in theory, develop software and build hardware.

Joys and Perils of Automation

Christos Cassandras spoke about automating the control of both physical and man-made systems. He provided some basic definitions for the middle school and high school audience, the essence of which are

- *System*: A group of some things working together toward a common purpose.

- *Control*: To exercise restraining or directing influence over; to regulate the system.

- *Automation*: An automatically controlled operation of a system to make the right decisions.

Christos showed block diagrams to illustrate the progression from uncontrolled system (Figure 5 (a)), to controlled system (Figure 5 (b)), to automatically controlled system (Figure 5 (c)). For example, suppose a tank of water with fixed height K is being filled from a pipe at the top with the pipe's flow rate controlled by a valve, with maximum flow rate of ρ. At time t, the tank is filled to height $x(t)$, which must be kept less than K. The valve allows water to flow into the tank at rate $u(t)$, where $0 \leq u(t) \leq \rho$. If the tank is initially empty, how do we fill it completely without overflowing by controlling $u(t)$? This control objective is the "desired behavior." (Figure 6 (a))

One control solution is to select the flow rate $u(t) = \rho$ until $t = K/\rho$ and $u(t) = 0$ after $t = K/\rho$ (Figure 6 (b)). What could go wrong? This solution works only if K and ρ are known precisely. In addition, the solution requires an accurate clock in the controller. Furthermore, suppose there were a leak in the tank or someone added water to the tank? This controller does not modify its operation if there are disturbances in the system, and the result in those cases would be an incorrect behavior.

Instead, we use feedback control to automatically control the tank to fill until it is full, then stop filling. The controller forces the flow $u(t) = \rho$ while $x(t) < K$, but $u(t) = 0$ when $x(t) = K$ can be implemented simply using a float connected to the valve. It is now becoming clear that we are modeling a flush toilet! The float blocks the input of water automatically

when the tank becomes full (Figure 6 (c)). This mechanism, which is the basis for how toilets work, is so effective it has remained unchanged for generations!

Automation has a cost, namely, the price of the float and related parts, but saves us from poor system performance when there is variability or disturbances. The resulting control $u(t)$ and the state $x(t)$ are the same as when the open-loop control is used, for the case when there are no disturbances on the system.

Our refined definition of automation is "putting together control and feedback." With this definition, some other examples of automation include the use of TCP/IP for Internet congestion control, autopilots in airplanes, and the control of train speed when approaching a station platform.

We can design controllers for physical systems or for human-made systems. Physical systems, for example the toilet, must satisfy natural laws, such as the conservation of mass. The values of the state of a physical system are generally real numbers and such systems are often time-driven. In contrast, human-made systems can be created with any rules that the designer selects. This choice can result in a real mess! An example of a human-made system is an ATM machine with a line of people waiting for service. In this system, the state, that is, the number of people waiting in the line, takes only integer values. Events, for example the arrival of a new person in the line or the push of a button, can change the state. Control protocols may take the form of the fair-play rule first-come-first-served. Such control rules, although fair to people, may conflict with priorities for maintenance or medical help, as in a hospital.

When we look for control solutions for these man-made systems, we find that computers can be rather dumb. Consider the example of a machine tool that operates on widgets. Suppose there is a buffer in front of a machine that can hold only a single part, and suppose the machine tool can only hold a single part. The machine will operate on the part and then either pass it on to an output area if the operation is successful or return it to the input buffer if it is defective. Unfortunately, if there is a waiting part in the input buffer, the machine tool will be stuck holding the defective part. Part 1 in the machine waits for the input buffer to be free, while part 2 in the buffer waits for machine to be free (Figure 7). This problem is called *deadlock*.

Here is a common *livelock* example involving people. Suppose that two people are approaching each other in a hallway. Both people both decide to move out of the way by stepping toward the side of the hallway. Unfortunately, they both step toward the same side, still blocking one another. Next, they both move toward the other side, still blocking each other. Have you had that happen to you? The same problem arises in computer buses when two computers try to talk on the bus at the same time. The computers must try to talk again at random times or else there will be livelock.

Similar man-made systems, where appropriate analysis and "science" is needed to solve the problems, include communication networks, manufacturing systems, traffic systems, and elevators, software systems, and video games. Christos showed the students photos of a computer-controlled LEGO car factory that he uses in his courses. Computer simulations are used to validate solutions that are designed before they are implemented on a real system. The cost of bringing down a manufacturing process for the update of the automation system is huge, in terms of loss of production, so any solution must be analyzed and validated thoroughly before it can be implemented.

Robots Playing Soccer

Raffaello D'Andrea described how university students under his supervision design and build small robots and the control and communication systems to support their operation. This is a very complex system. The robots play a scaled-down version of soccer, where the balls are close in size to golf balls (Figure 8). The game is played according to regular soccer rules; for example, holding is not allowed. The robots must play autonomously with no human involvement once the game begins. Student-designed robotic soccer teams compete in national and international tournaments, leading up to the RoboCup world championships.

The control structure here is much more complex than those described above. Hierarchical control is used, separating local control tasks for robot operation from the centralized planning required for the robot team's play strategies. A vision system is implemented so that the information is available not only on your team's robots' locations, but also on the location and behavior of the other team's robots and the ball. A communication system is needed to pass information between the vision system, the centralized controller, and the localized robot controllers. Physical design decisions included robot size, acceleration, speed, maneuverability, and inertial navigation. Low latency (meaning short time delay) was critical in the network design.

Other considerations included the following. For the local robot controller design, the robot-motor system dynamics had to be understood. Motion planning for a single robot included path planning for obstacle avoidance and to ensure that the robot was where it is needed when it was needed there. High level "plays" included both offensive and defensive strategies, and transitions between these as appropriate. The time required for a play must be kept small in certain situations. Otherwise, the energy used had to be minimized. This determined the choice of control actions and trajectories selected. Algorithms are tested thoroughly using simulated games before they are implemented.

Video fragments of the international competitions where the Cornell Autonomous Robotic Soccer Team participated were proof of the design and were highlights of the presentation (Figure 9). The Cornell team won four championships in the last five years. Students asked how the robots worked and what the student designers did. One of veteran teachers of these high school workshops, Lynette Forinety, said she would like to use the robotic soccer videos for classroom presentations, and asked how she could get them. The competition highlight videos are available on Raffaello's web site (www.mae.cornell.edu/raff).

Autonomous Vehicles: Racing from Los Alamos to Las Vegas

Richard Murray described this grand research challenge to the students. This is a DARPA challenge, and Richard captured the students attention when he mentioned the $1 million award. On 3/13/04, weather permitting, fully autonomous vehicles will race across the desert with no humans in the loop (no drivers, and no remote operation by humans allowed). To win the prize, the vehicle must complete the trip from Barstow, California, to Las Vegas, Nevada, in 10 hours or less. Two hours before the race, participants will be given a set of 1000 GPS points and a corridor 10 meters wide to 10 km wide to stay within. They will not

be told where in that corridor the path is that can be driven successfully in a 4-wheel drive. The vehicle also must not run into any other vehicles, Joshua trees (which are protected plants), fences, and other obstacles. The race is a 250-mile run with dirt roads, trails, rough roads, open desert, lake bed, overpasses to go under, water crossings, and dead ends.

For professional desert human racers with prior experience on that course, the trip would take 4 hours. For same without prior experience on that route, the trip would take 8 hours. How can we do this in 10 hours or less without any human? Murray showed photos of many vehicles that are attempting the race. These vehicles are full-scale in size. They may be standard 4 wheel drive vehicles, or specially designed racing vehicles.

Richard also gave some more information about his team's vehicle (Figure 10). The sensors consist of 12 cameras for vision, a GPS navigation system, IMU, LADAR (laser radar scanning), and other devices. For the actuation, the driver's seat entirely replaced with automated parts. For the computation, there were 10 computers. Not only does the vehicle have to drive on its own through the desert without hitting anything, it also has to determine the proper path through difficult and confusing terrain.

He also showed us footage taken from professional off-road racers traversing the course. The videos taken from the vehicle hurdling through the desert keeping to an ill-defined path fascinated the students and lead to a lot of questions. For example, one poignant question that was asked was

Question: "If the truck kills someone, are you liable for murder?"

Better Movies through Mathematics

Brian Rosen described how animated movies are made and about the importance of mathematics in animation. We learned that effects are possible in animated movies that are not possible in reality, for example a light can change colors depending on the object it's hitting. Since the animated world is defined mathematically, artists are free to explore looks that would be very difficult to obtain using only the laws of physics.

"Art comes first."

Brian emphasized that the artists' renditions of the characters drove the mathematical modeling and animation, first in how the characters should look, later in the range of facial expressions they should have, and finally in how the light and shadows should appear in each scene.

Once the artist has provided sketches of the characters, they must be represented mathematically so that they can be manipulated using the animation software. Much of the mathematics of visualization can be described a level understandable to high school students. Brian showed the students many different ways to represent a line, parameterized in terms of a start point (x_0, y_0) and an end point (x_1, y_1) with t as the line parameter with $0 \leq t \leq 1$. He eventually derived the parametric form of the line $(X, Y) = (X(t), Y(t))$ in both coordinates:

$$X(t) = x_1 \cdot t + x_0 \cdot (1 - t),$$
$$Y(t) = y_1 \cdot t + y_0 \cdot (1 - t).$$

Since animated characters are not composed of straight-line segments, Brian next showed the students how to represent various curves is in terms of 3rd degree polynomials with general coefficients $\{c_0, c_1, c_2, c_3\}$. The line

$$X(t) = c_1 \cdot t + c_0$$

is described by two points, the parabola

$$X(t) = c_2 \cdot t^2 + c_1 \cdot t + c_0$$

is described by 3 points, and the cubic (3rd degree polynomial)

$$X(t) = c_3 \cdot t^3 + c_2 \cdot t^2 + c_1 \cdot t + c_0$$

is described by four points. Knowing only the start and end points will not be sufficient to determine the constants c_i for $i = 0, 1, 2, 3$, for the cubic. Other information, such as derivative information, is required. However, we can write the derivative with respect to the parameter t for the cubic as

$$X'(t) = 3 \cdot c_3 \cdot t^2 + 2 \cdot c_2 \cdot t + c_1.$$

Next <u>splines</u> of curves are used to glue together the pieces described by 3rd degree polynomials, since splines rely on both point and derivative information to give smooth transitions between the spline pieces. In three dimensions, we use three 3rd-degree polynomials, one for each dimension, to form surfaces of a figure of interest. The figure is represented by meshes, which are sets of neighboring points on the surface of the figure. We move the underlying surface mesh to get motion and more realism in animated films and games.

This provides the animation!

The animation department is responsible for creating the movement of the characters, and it is crucial that the meshes created by the technical department be capable of every shape necessary to tell the story. The art department often sketches the characters in a multitude of expressions, and the animators test the three dimensional meshes by trying to match each one of the expressions. Sometimes human refernece subjects are video taped to help the animators analyze the subtleties of motion.

If we stopped here, Brian pointed out that the characters would still look flat. A sphere the same color everywhere looks like a solid disk if there are no lighting effects. He explained how mathematics is used to model how light interacts with surfaces to give objects the appearance of being three-dimensional. For ideal surfaces, the reflection is specular with a uniquely defined direction. This interaction for real surfaces when the object is not smooth and has imperfections is even more complex. We need to use the <u>dot product</u>, which is taught in high school math! Diffuse light reflection is proportional to dot product ($\bullet$) of the normal ($\vec{N}$) to sphere and the direction ($\vec{L}$) of the source of light:

$$\text{Diffuse} = \vec{N} \bullet \vec{L}.$$

However, the eye of the viewer also receives specular or directly reflected light such that

$$\text{Brightness} = \text{Specular} + \text{Diffuse}.$$

The specular model that Pixar wrote includes a color shaping effect that allows for iridescent effects. In reflection, the color comes from the environment and color shape reflection.

Another technique is procedural displacement using sine waves and random noise to simulate more realism.

To get realistic lighting effects, there are teams of people adjusting computer light sources. These lighting teams are inspired by pastels of the scenes created by artists. The lighting is not limited to a certain number of sources, or lights in certain physical locations. The teams can put lights places that filmmakers could not in real life! In addition, they can create lights in animation that shine on part of the scene. Even negative lighting is possible, where certain light sources remove light, rather than adding it, to the scene.

Production of an animated movie takes 3 years per movie, one year for concept development and two years for production, with roughly 200 people working on the movie during production.

Question: How long did it take to learn the math?

Answer: Brian said he learned computer graphics in his sophomore year of college. Three years later, in 1993, he started a job in animation. He said he learned a lot on the job, too.

Later in the panel discussion, Brian answered more questions and gave extended explanations of the role of mathematics: the important role that algebra, trigonometry, geometry and other areas of mathematics play in scientific visualization and animation. Computer animation also plays an important role in computer games, another topic of interest to students. There were many particular questions on the making of particular animated films and the possibility of sequels to existing films.

It is clear that this talk made a convincing argument of the necessity of learning mathematics well enough to be able to apply it in science and engineering. Some of these questions led to more general questions in the later session about careers and college: the cost of college, relative benefits of public versus private colleges, and what you do in college. Students also wanted to know if you needed to decide your major before starting college, or if you could wait and discover what major you would like while in college.

Understanding and Controlling the Pendulum

Katsuhisa Furuta of Tokyo Denki University showed the students how pendulums could be controlled to stand upright, both with videos and equations.

Galileo Galilei long ago investigated the periodic swinging motion of a hanging pendulum. Today, an extreme challenge is to keep the pendulum inverted, much like holding an umbrella upward with the handle on the palm. The problem is how to exert control to counter-balance the downward pull of gravity. Video examples were shown of a single pendulum balanced by means of a single unattached link to a robot arm using feedback control and vision system sensors.

An inverted single pendulum is now a standard demonstration at conferences and in classroom laboratories. Two separate single-link pendulums seated on the same base appear to be an impossible task since they can not be controlled separately and may have different disturbances. However, they can be inverted simultaneously without feedback control using

very fast vertical vibrations on the base.

The single-link pendulum, hanging or inverted, satisfies the equation

$$J \cdot \theta'' = -m \cdot g \cdot l \cdot \sin(\theta),$$

where m is the pendulum mass, l is the pendulum length, g is the acceleration of gravity, $J = m \cdot l^2$, θ is the angle measured from the downward vertical axis, and θ'' is the second time derivative or angular acceleration. For the inverted pendulum, let $\theta = \pi - \psi$, so that $\psi = 0$ corresponds to the inverted pendulum pointing upward. For the inverted pendulum, it is easier to use the equation

$$J \cdot \psi'' = +m \cdot g \cdot l \cdot \sin(\psi).$$

To have the pendulum swing upward without a problem, we would like to make the pendulum swing upwards as if gravity itself were reversed. We would like our equation to be

$$J \cdot \psi'' = -m \cdot g \cdot l \cdot \sin(\psi)!$$

How do we do this? We can not change the actual gravitational force of earth, but we can apply a new force to the pendulum.

Adding a control term to the actual pendulum equation leads to

$$J \cdot \psi'' = +m \cdot g \cdot l \cdot \sin(\psi) - u \cdot m \cdot l \cdot \cos(\psi),$$

where u is the control variable. The control (meaning the equation for u in terms of ψ) that makes the right-hand-side of the equation take the form of the pendulum with gravity upwards towards heaven rather than downwards is

$$u = u(t) = 2g \cdot \sin(\psi)/\cos(\psi),$$

Using this controller, we have added a force that makes the pendulum behave as if gravity were reversed.

More videos showed the double-link pendulum being stabilized after the double pendulum is swung up. Even more extraordinary, the triple pendulum was shown stabilized (Figure 11 (a)). Finally, Katsuhisa showed "cooperative control", the hand-off of an inverted pendulum between one robot and another (Figure 11 (b)). The coordination of the robots is the hard part.

Katsuhisa also told the students about the important and dangerous job of clearing land mines in war-ravaged parts of the world and how that job was going to be automated to protect the humans better. He is involved in the automation effort. This resonated with the students, because a nearby Hawaiian island that had been used for U.S. military target practice has recently been cleared of unexploded ordinance. There are 10 million buried mines in Afghanistan alone, and many more in Cambodia, Vietnam, and Laos. The hand clearing of mines needs to be replaced by machines to clear mines more safely, and Japanese engineers are helping.

Future Careers in Embedded Systems, Mechatronics, and Control

Mark Spong spoke on embedded systems and mechatronics, the hybrid combination of mechanics and electronics, and what problems could be tackled in that field.

Cell phones have computers in them, so they are called embedded systems! Embedded systems link together different devices emerging from the Information-Technology (IT) revolution. Mechatronics means putting intelligence on physical systems, mechanical systems plus electronics including sensors, actuators, computers, software and intelligence.

Examples of mechatronics include automotive systems, adaptive cruise control, conversion of gas engines to hybrid and fuel cell engines, drive-by-wire, and cam-less engines.

In a traditional car, the car is turned when the steering wheel is turned. There are various mechanical linkages in this arrangement. The steering wheel is a danger to the driver in an accident. If the steering column and mechanical connections are replaced by sensors, actuators, and embedded microprocessors, then not only is the steering column eliminated, but also a left-hand drive car can be easily converted to a right-hand drive car.

In a traditional engine, a mechanical cam pushes the rods as it turns to open and close the valves. The cam is a certain shape, and that dictates the timing of the valve operation. If motors are used to open and close valves instead, then complete control of timing is possible. Some of the pistons can be stopped at traffic lights, when the extra thrust is not needed.

Robots are the ultimate mechatronic systems. They enable telematics, unmanned vehicles and telemedicine. Robots permit automation of the manufacturing process. Some robots are even humanoid in form. Mark showed a video showing a robot, developed by his students, which played air hockey against a human opponent. The ending of the video, when the robot lost control after an unusual maneuver, prompted a series of questions.

How to Turn a Single Dollar into Billions

Shane Haas gave a talk on how the timing of investments could be used to maximize return on investments (if anyone had perfect a priori information!). He is a Vice-President of the financial hedge fund start-up company AlphaSimplex Group in Cambridge, Massachusetts, and a recent electrical engineering Ph.D. with a minor in finance from MIT. A surprising number of the students were knowledgeable about finance and a few owned stock and so had a personal interest in his talk. He described an investment approach where an investor moved his or her entire investment, at regular intervals, to the best instrument for the next period of time. These could alternate between a savings account with a low fixed rate of return, and a volatile investment such as a stock fund. The portfolio instant rate of return is

$$P(t) = \text{maximum}[B(t), S(t)],$$

where $B(t)$ is the risk-less fixed rate of return and $S(t)$ is the risky stock fund rate of return.

Shane used monthly closing data for the period from 1926-2003, updating the portfolio each month according to the formula, for U.S. Treasury bills as low-risk investment and the

Standard and Poor's 500 stock index as the high-risk investment (simulating a large stock mutual fund). Starting from one dollar in 1926, the final amount was an amazing $14.3 billion! If one only invested one's money in S&P 500 stock index, the $1 would have only become $2171 over that time. In a savings account with a low fixed interest rate, it would become only $17.

This perfect market timing investment strategy, that is, switching between the better of S&P 500 or Treasury bill returns, is equivalent to buying treasury bills and a call option on the S&P 500. Remarkably, the Black-Scholes formula can price this option to determine the value of this investment strategy. This analogy motivated a question from our student financial experts about investment or transaction fees, and Shane said that according to the Black-Scholes formula, the fees justified for someone with perfect monthly prediction would be 24% of the investment on an annual basis.

Of course, this assumes perfect market timing! If perfectly switched between those two on a daily basis, then it would be far more than $14 billion with perfect timing.

He also suggested a better alternate market timing portfolio,

$$P(t) = \text{maximum}[\text{Intel}(t), \text{Microsoft}(t)],$$

where the Intel(t) and Microsoft(t) are the daily returns. With daily perfect market timing, one dollar invested on January 5, 1990, would become $2.6 trillion on December 5, 2003. Shane encouraged students to download historical data from *finance.yahoo.com* and try other stocks.

Of course, we would not recommend that students try timing the market with their own money. No one has perfect timing and the minimum investment in a hedge fund to have a professional financial analyst do the timing would be about $1 million to $25 million; too much for these students. Pension funds, wealthy individuals, and endowment funds typically invest in hedge funds.

Mathematical Games

Suzanne Lenhart of the University of Tennessee and Past President of the Association for Women in Mathematics introduced some mathematical games to close the day. The students warmed up by playing Hex (a positional strategy two-player game), and then conducted some probabilistic experiments with number-spinners relating to the well-known rock–paper–scissors game. By playing these games, the students learned about the property of transitivity. (For an example of non-transitivity, if spinner P beats spinner R with probability 5/9 and spinner R beats spinner S with probability 5/9, it is not true that spinner P would beat spinner S.)

Acknowledgments

SIDEBAR: Outrigger - The Feedback Mechanism That Allowed the Polynesians to Colonize the Pacific

During a History Session at CDC 2003, Danny Abramovitch presented a plenary paper showing that the outrigger (from an outrigger canoe) (Figure 1) may in fact be humanity's first feedback mechanism. The Polynesian migration into the Pacific dates back to about 3500 years ago and this would have been impossible in dugout canoes without outriggers. Thus, the outrigger was one of the technologies that enabled the Polynesians to colonize the Pacific. At its most basic level, an outrigger consists of some sort of a float, attached via one or more booms to the gunwales (top edge) of a boat (Figure 2). While modern outriggers can be made from a variety of sturdy, buoyant materials, the outrigger float has traditionally been some piece of light wood, and the type of boat that it is most often associated with is a dugout canoe. Qualitatively, the operation of the outrigger is rather simple. When the canoe rotates so as to raise the float from the water, its weight at the end of a boom provides torque to rotate the float back to the surface. When the rotation of the canoe acts to push the float into the water, its buoyancy acts to restore the float to the surface of the water. Thus, the outrigger dramatically increases the roll stability of the small canoes to which they are typically attached.

SIDEBAR: NSF Research and K-12 Education — A Winning Partnership

NSF Program Director Kishan Baheti gave an overview of NSF's mission to support fundamental research and education in science and engineering. Dr. Baheti highlighted NSF's role in the emerging field of nanotechnology that will lead to unprecedented understanding and control over the fundamental building blocks of all physical things. This is likely to change the way we design and manufacture computers, vaccines, automobiles and many other products. To encourage innovations, he described NSF's Research Experience for Teachers (RET) Program, which supports a primary or secondary teacher's direct experience in a research laboratory or similar research environment. The program depends on a teacher connecting up with a researcher, who can supplement an existing NSF grant or initiate a new one to obtain teacher support.

Researchers are encouraged to involve teachers in funded research. This program encourages transfer of new knowledge to pre-College classrooms and provides support for both teacher enhancement and the development of educational materials for the K-12 classroom.

SIDEBAR: Past Speakers and Topics

This series of workshops has benefited not only from this group of exceptional speakers, but from others as well. In past workshops, researchers in control system and mathematics gave the following talks:

- "Using Mathematics for Epileptic Seizure Warning," Ivan Osorio, M.D., Director, Comprehensive Epilepsy Center University of Kansas Medical Center, and Mark Frei, Ph.D., Operating Manager and Technical Director, Flint Hills Scientific, L.L.C.

- "Music, the Brain, and Complex Adaptive Systems: Using Musical Metaphors and Models to Study Brain Functions," Bryan Haaheim University of Kansas, and Deron McGee, University of Kansas.

- "Brownian Motion: Past and Present," Tyrone E. Duncan, University of Kansas.

- "How the Internet and Wireless Networks are Controlled: What's Happening behind the Scene," P. R. Kumar, University of Illinois at Urbana-Champaign.

- "Control of Jet Engines," Richard M. Murray, California Institute of Technology.

- "Powerful Ideas in the World of the Child," Alan Kay, Disney Fellow and Vice President of Research and Development, The Walt Disney Company.

- "Making Calculus Fun: How to Entertain at Parties," Colin Adams, Williams College.

- "Systems, Control, and Mathematics," Stephen Boyd, Stanford University.

- "How Feedback Changed the World," Dennis Bernstein of the University of Michigan. This talk resulted in the articles "Feedback Control: An Invisible Thread in the History of Technology," *IEEE Contr. Sys. Mag.*, Vol. 22, pp. 53-68, April 2002, and "Introducing Signals, Systems, and Control in K-12," *Contr. Sys. Mag.*, Vol. 23, pp. 10-12, April 2003.

SIDEBAR: Comments from Students about the Workshop

Numerous rewarding comments were received from participants after the workshop. Among the many received, here are a few.

- "What a wonderful day. My students were all greatly impressed with the level of knowledge presented, the achievements of the young students, and the many ways that math could be applied. Of course, the soccer team was the favorite, but among my students it seems that most of the presentations were a favorite with a smaller group of students. Education is a very hard sell so the kind of exposure you offered our students was most welcome. Thank you again for the invitation."

- "It was wonderful. The students had wonderful things to say about the speakers we heard. Thank you for taking the time to arrange this very worthwhile event for us."

- "The trip to the Workshop had a lot of information that at first seemed to not appeal to a middle or high school student. After a short time, with the excellent treatment from the hosts, it promised to be a beneficial experience. I enjoyed, and found interesting, all of the presentations, although some were beyond the middle or even high school level. When presenters were lecturing students at the high school level, we could immediately see the wonders of what they were talking about to us. We saw everything from futuristic cars to ingenious displays of mechatronics. We could instantly see the application to our education and what there is in the future. Of course, being interested in math and science, we could even imagine ideas of fascinating creations beyond what was presented."

- "The workshop was the most memorable and successful event. It was truly a collaborative effort of so many people."

- "I do not know how to find the words for how wonderful my trip was–I learned SO much and am forever grateful. As I write this, I am also working on a number of lesson plans based on the information I learned. You did an amazing job with the full-day workshop. I know how much work those are and have organized them myself. On the whole, I felt that the students took away a lot of newly found enthusiasm for engineering. Now, that enthusiasm has to be cultivated by their teachers. I am working on condensing my notes into an outline form and will email it to you once I am done. Feel free to share it with the other teachers if you'd like to. I would love to help you put on your next one so please, keep me in mind! I could certainly put together some pre and post activities and assessments and would enjoy doing it."

SIDEBAR: Comments from the Maui District School Superintendent

The following comments were received from Allen Ashitomi, Maui District School Superintendent:

> Dr. Bozenna Pasik-Duncan, I am glad you received such an overwhelming response. I must apologize for not being able to offer more assistance, but I have only a minimal staff and we are working hard on implementing requirements of the Federal No Child Left Behind law, and our State's standards-based education initiatives. I was on my computer past midnight last night just responding to email. There just aren't enough hours in the day. I am really interested in science though, and I believe we need to develop that component of our curriculum. I started in electrical engineering and then switched to education, but my heart is in the sciences. I am intrigued with the developments in genetics and astronomy. I'm still trying to understand string theory. Do you have a simple explanation? I again want to thank you for offering this opportunity to students in our school system.

SIDEBAR: Bozenna Pasik-Duncan Receives Louise Hay Award for Contributions to Mathematics Education

Temporary Substitute

In recognition of her wide-range of outstanding work as a mathematician, the Association for Women in Mathematics presented the Louise Hay Award to Bozenna Pasik-Duncan of the Department of Mathematics at the University of Kansas.

Bozenna Pasik-Duncan is a research mathematician with a deep commitment to education. She has been recognized for her teaching from the time she was a Lecturer in Warsaw, when she received the National Teaching Award from the Ministry of Higher Education and Sciences. At Kansas, she has continued to receive several teaching awards, including one for distinguished teaching and the profound impact made on students' lives, and another awarded by graduating seniors.

But Pasik-Duncan's work in education extends beyond her exceptional skill as a teacher. In nominating her, Professor Jack Porter, Chair of her department, said that her philosophy is that every student from high school senior to undergraduate to graduate will experience research that bridges mathematic with different fields (for example, biology, physics, chemistry, economics, and medicine). Pasik-Duncan has worked to make this vision come alive. Through the Research Experiences for Undergraduates Program of the National Science Foundation (NSF), Professor Pasik-Duncan has, since 1992, mentored students and nurtured them in their studies. Indeed, her NSF Control Workshops enhance the connection among high school students, mathematics and science teachers, and research groups in Control Systems. As a co-investigator of projects supported by NSF and the Sprint Corporation, she involved her graduate students in real world applications of control.