Computational Stochastic Multistage Manufacturing Systems with Strikes and Other Adverse Random Events

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Abstract

Multistage manufacturing systems (MMS) are models for the assembly of consumable goods. In the simple case, a linear assembly line of workstations, components, or value, are added to the product. Some examples assembly line products are automobiles or printed circuit boards. Production scheduling typically takes in to account workstation repair, failure, and defective pieces as stochastic events, effecting the workstation production rates. The supply routing problem of raw materials is not usually taken into account. However, in this treatment, the effects of strikes and natural disasters, which may affect the routing of raw materials, are considered for the MMS. Numerical results illustrate the optimal control of MMS undergoing strikes, as well as workstation repair and failure.

1. Introduction

In this paper, multistage manufacturing systems (MMS) are considered for the assembly of a single consumable good. The sequence of stages necessary to complete the finished good is represented as a linear chain of stages at which a subcomponent or value is added. Each stage consists of a number of workstations which are assumed to be identical in all respects and operate at the same level. The workstations are subject to repair and failure. The control model for the production scheduling problem needs to account for these stochastic events in order to insure that the production goal is met. The discipline assumed for the MMS is that of *Just in Time* or *Stock less Production* which does not require that large inventories of raw materials be kept on hand (see Hall [6]). Bielecki and Kumar [2] show that a zero inventory policy is optimal for a manufacturing system that is subject to uncertainties, this further justifies the *Just in Time* manufacturing discipline.

The goal of the control problem is to account for all stochastic events, such as workstation repair and failure, strikes, and natural disasters, so that the production goal is achieved in a specified way. The cost functional is used to impose penalties for shortfall or surpluses in production while maintaining a minimum control effort discipline. In this model, strikes and natural disasters can affect the MMS directly or the way in which raw materials enter the MMS, and therefore can limit the throughput of the MMS if there is not sufficient raw materials which is the *routing problem*. This consideration is of great importance since strikes and natural disasters can have very significant impact on the financial well being of a company. These concepts which are features of the model presented here are illustrated by the United Auto Workers strikes against General Motors [1, 8], the strike of the United Parcel Service [10, 11], and natural disasters such as earthquakes and floods, for example.

The model considered here is an extension of the production scheduling model by Westman and Hanson [18] which utilizes state dependent Poisson processes [16] to model the rare events of workstation repair and failure as well as strikes and natural disasters. In [18], all catastrophic events, strikes and natural disasters, that effect the MMS are lumped together in one term. This allows the model to be a lower dimension, but presents a problem with tracking the catastrophic events and there resolution. In this treatment, each of the catastrophic events is represented by its own state variable so that complete tracking of the event history can be accurately represented. Additionally, this model includes temporal consideration for the effects of the strikes and natural disasters on the demand rate, which is viewed as a constant in [18]. This feature of the model is necessary in order to redefine the way in which the production goal is to be met dependent on whether a strike occurs or not. The production scheduling in this model *anticipates* catastrophic

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events and compensates for them, however this may not be enough (dependent on the length of the strike) or too much if a strike does not occur. Therefore the demand rate may need to be dynamically adjusted.

For completeness the *LQGP problem* (see [13]) using extensions for state dependent Poisson noises (see [16, 18]) is presented in Section 2.. The model for the production scheduling of a MMS presented here forms a *LQGP problem* utilizing state dependent Poisson noises and a rebalancing of the demand rate is given in Section 3. In Section 4. two examples of state path realizations are presented that show the effects of a strike in conjunction with demand rate rebalancing.

2. LQGP Problem Formulation

For completeness we present the canonical form for the LQGP problem that originally appears in Westman and Hanson [13], for the case with state independent Poisson noise, and [16] for state dependent Poisson noise. Additionally, considerations for modeling a physical system are presented as well, as well as formal solution to the LQGP problem.

The linear dynamical system for the LQGP problem is governed by the stochastic differential equation (SDE) subject to Gaussian and state dependent Poisson noise disturbances is given by

$$\begin{aligned} d\mathbf{X}(t) &= [A(t)\mathbf{X}(t) + B(t)\mathbf{U}(t) + \mathbf{C}(t)]dt \\ &+ G(t)d\mathbf{W}(t) + [H_1(t)\cdot\mathbf{X}(t)]d\mathbf{P}_1(\mathbf{X}(t),t) \\ &+ [H_2(t)\cdot\mathbf{U}(t)]d\mathbf{P}_2(\mathbf{X}(t),t) \\ &+ H_3(t)d\mathbf{P}_3(\mathbf{X}(t),t), \end{aligned} \tag{1}$$

for general Markov processes in continuous time, with $m\times 1$ state vector **X**(t), $n\times 1$ control vector **U**(t), $r\times 1$ Gaussian noise vector **dW**(t), and $q_\ell\times 1$ space-time Poisson noise vectors $d\mathbf{P}_\ell(\mathbf{X}(t),t)$, for $\ell=1$ to 3. The dimensions of the respective coefficient matrices are: $A(t)$ is $m\times m$, $B(t)$ is $m\times n$, $\mathbf{C}(t)$ is $m\times 1$, $G(t)$ is $m\times r$, while the $H_\ell(t)$ are dimensioned, so that

$$[H_1(t)\cdot\mathbf{x}]=\left[\sum_k H_{1ijk}(t)x_k\right]_{m\times q_1}, \tag{2}$$

$$[H_2(t)\cdot\mathbf{u}]=\left[\sum_k H_{2ijk}(t)u_k\right]_{m\times q_2}, \tag{3}$$

and

$$H_3(t) = [H_{3ij}(t)]_{m\times q_3}. \tag{4}$$

Note that the space-time Poisson terms are formulated to maintain the linear nature of the dynamics, but the first two are actually bilinear in either $\mathbf{X}$ or $\mathbf{U}$ and $d\mathbf{P}_\ell$ for $\ell=1$ or 2, respectively.

The *state dependent Poisson noise* can be viewed as a sequence of events that is represented by its ith couple

$$\{T_i(\mathbf{X}(T_i)),\ M_i(\mathbf{X}(T_i))\}, \tag{5}$$

for $i=1$ to k, where $T_i(\mathbf{X}(T_i))$ is the time for the occurrence of the ith jump with state dependent mark amplitude $M_i(\mathbf{X}(T_i))$. This representation of the Poisson process provides more realism and flexibility for a wider range of stochastic control applications since the arrival times and amplitudes may depend of the state of the system. Additionally, this formulation allows for simpler dynamical system modeling of complex random phenomena.

The state dependent vector valued marked Poisson noises are related to the Poisson random measure (see Gihman and Skorohod [5] or Hanson [7]) and are defined as

$$\begin{aligned} d\mathbf{P}_\ell(\mathbf{X}(t),t) &= [dP_{\ell,i}(\mathbf{X}(t),t)]_{q_\ell\times 1} \\ &= \left[\int_{\mathcal{Z}_{\ell,i}} z\mathcal{P}_{\ell,i}(dz,\mathbf{X}(t),dt)\right]_{q_\ell\times 1} \end{aligned} \tag{6}$$

for $\ell=1$ to 3 which consists of q_ℓ independent differentials of space-time Poisson processes that are functions of the state, $\mathbf{X}(t)$, where z is the Poisson jump amplitude random variable or the mark of the $dP_{\ell,i}(\mathbf{X}(t),t)$ Poisson process where $\ell=1$ to 3 and $i=1$ to q_ℓ. The mean or expectation is given by

$$\begin{aligned} &\mathrm{Mean}[d\mathbf{P}_\ell(\mathbf{X}(t),t)] \\ &= \Lambda_\ell(\mathbf{X}(t),t)dt\int_{\mathcal{Z}_\ell}\mathbf{z}\phi_\ell(\mathbf{z},\mathbf{X}(t),t)d\mathbf{z} \\ &\equiv \Lambda_\ell(\mathbf{X}(t),t)\overline{\mathbf{Z}}_\ell(\mathbf{X}(t),t)dt, \end{aligned} \tag{7}$$

where $\Lambda_\ell(\mathbf{X}(t),t)$ is the diagonal matrix representation of the state dependent Poisson rates $\lambda_{\ell,i}(\mathbf{X}(t),t)$ for $\ell=1$ to 3 and $i=1$ to q_ℓ, $\overline{\mathbf{Z}}_\ell(\mathbf{X}(t),t)$ is the mean of the jump amplitude mark vector and $\phi_{\ell,i}(z,\mathbf{X}(t),t)$ is the density of the (ℓ,i)th amplitude mark component. Assuming componentwise independence, $d\mathbf{P}_\ell(\mathbf{X}(t),t)$ has covariance given by

$$\begin{aligned} &\mathrm{Covar}[d\mathbf{P}_\ell(\mathbf{X}(t),t),d\mathbf{P}_\ell^\top(\mathbf{X}(t),t)] \\ &= \Lambda_\ell(*)dt\int_{\mathcal{Z}_\ell}(\mathbf{z}-\overline{\mathbf{Z}}_\ell(*))(\mathbf{z}-\overline{\mathbf{Z}}_\ell(*))^\top\phi_\ell(\mathbf{z},*)d\mathbf{z} \\ &\equiv \Lambda_\ell(*)\sigma_\ell(*)dt, \end{aligned} \tag{8}$$

with, for instance, $\sigma_\ell(*)=\sigma_\ell(\mathbf{X}(t),t)=[\sigma_{\ell,i,j}\delta_{i,j}]_{q_\ell\times q_\ell}$ denoting the diagonalized covariance of the amplitude mark distribution for $d\mathbf{P}_\ell(\mathbf{X}(t),t)$. Again, the mark vector is not assumed to have a zero mean, i.e., $\overline{\mathbf{Z}}_\ell\neq 0$, permitting additional modeling complexity. Note, that for discrete distributions the above integrals need to be replaced by the appropriate sums.

The Gaussian white noise term, $d\mathbf{W}(t)$, consists of r independent, standard Wiener processes $dW_i(t)$, for $i=1$ to r. These Gaussian components have zero infinitesimal mean,

$$\mathrm{Mean}[d\mathbf{W}(t)]=\mathbf{0}_{r\times 1} \tag{9}$$

and diagonal covariance,

$$\text{Covar}[d\mathbf{W}(t), d\mathbf{W}^T(t)] = I_r dt. \quad (10)$$

It is further assumed that all of the individual component terms of the Gaussian noise are independent of all of the Poisson processes,

$$\text{Covar}[d\mathbf{W}(t), d\mathbf{P}_\ell^T(t)] = 0_{r\times q_\ell}, \quad (11)$$

for all ℓ.

The jth jump of the $\{\ell, i\}$th space-time Poisson process at time $t_{\ell,i,j}$ with amplitude $M_{\ell,i,j}$ causes the following jump from $t^-_{\ell,i,j}$ to $t^+_{\ell,i,j}$ in the state:

$$[\mathbf{X}](t_{\ell,i,j}) = \left\{ \begin{array}{ll} [H_1(t_{\ell,i,j})\mathbf{X}(t^-_{\ell,i,j})]_i M_{\ell,i,j}, & \ell = 1 \\ [H_2(t_{\ell,i,j})\mathbf{U}(t^-_{\ell,i,j})]_i M_{\ell,i,j}, & \ell = 2 \\ [H_3(t_{\ell,i,j})]_i M_{\ell,i,j}, & \ell = 3 \end{array} \right\}. \quad (12)$$

From the above statistical properties of the stochastic processes, $d\mathbf{W}$ and $d\mathbf{P}_\ell$, it follows that the first two conditional infinitesimal moments of the state, fundamental for modeling applications, are

$$\begin{aligned} & \text{Mean}[d\mathbf{X}(t) \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \\ = & \big[A(t)\mathbf{x} + B(t)\mathbf{u} + \mathbf{C}(t) + [H_1(t)\mathbf{x}](\Lambda_1\overline{\mathbf{Z}}_1)(\mathbf{x}, t) \\ + & [H_2(t)\mathbf{u}](\Lambda_2\overline{\mathbf{Z}}_2)(\mathbf{x}, t) + H_3(t)(\Lambda_3\overline{\mathbf{Z}}_3)(\mathbf{x}, t)\big]\, dt \quad (13) \end{aligned}$$

and the conditional infinitesimal covariance,

$$\begin{aligned} & \text{Covar}[d\mathbf{X}(t) \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \\ = & \big[(GG^T)(t) + [H_1(t)\mathbf{x}](\Lambda_1\sigma_1)(\mathbf{x}, t)[H_1(t)\mathbf{x}]^\top \\ + & [H_2(t)\mathbf{u}](\Lambda_2\sigma_2)(\mathbf{x}, t)[H_2(t)\mathbf{u}]^\top \\ + & H_3(t)(\Lambda_3\sigma_3)(\mathbf{x}, t)H_3^\top(t)\big]\, dt. \quad (14) \end{aligned}$$

The quadratic performance index or cost functional that is employed is quadratic with respect to the state and control costs, is given by the *time-to-go* or *cost-to-go* functional form:

$$V[\mathbf{X}, \mathbf{U}, t] = \frac{1}{2}(\mathbf{X}^\top S\mathbf{X})(t_f) + \int_t^{t_f} C(\mathbf{X}(\tau), \mathbf{U}(\tau), \tau)d\tau \quad (15)$$

with

$$C(\mathbf{x}, \mathbf{u}, t) = \frac{1}{2}\left[\mathbf{x}^\top Q(t)\mathbf{x}, + \mathbf{u}^\top R(t)\mathbf{u}\right] \quad (16)$$

where the time horizon is (t, t_f), with $S(t_f) \equiv S_f$ is the quadratic final cost coefficient matrix and $C(\mathbf{x}, \mathbf{u}, t)$ is quadratic instantaneous cost function. The final cost, known as the *salvage cost*, is given by the quadratic form,

$$\mathbf{x}^\top S_f\mathbf{x} = S_f : \mathbf{x}\mathbf{x}^\top = \text{Trace}[S_f\mathbf{x}\mathbf{x}^\top]. \quad (17)$$

In order to minimize (15) requires that the quadratic control cost coefficient $R(t)$ is assumed to be a positive definite $n \times n$ array, while the quadratic state control coefficient $Q(t)$ is assumed to be a positive semi-definite $m \times m$ array. The coefficients $R(t)$ and $Q(t)$ are assumed to be symmetric for simplicity. The LQGP problem is defined by (1, 15).

The stochastic dynamic programming approach is used to solve the control problem. So a functional, the *optimal, expected cost*, is defined as:

$$v(\mathbf{x}, t) \equiv \underset{\mathbf{u}[t,t_f)}{\text{Min}} \left[\underset{\mathbf{P},\mathbf{W}[t,t_f)}{\text{Mean}} [V \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{U}(t) = \mathbf{u}] \right], \quad (18)$$

where the restrictions on the state and control are that they belong to the admissible classes for the state, $\mathcal{D}_\mathbf{x}$, and control, $\mathcal{D}_\mathbf{u}$, respectively. A final condition on the optimal, expected value, is determined from the final or *salvage* cost using (18) with $V[\mathbf{X}, \mathbf{U}, t_f]$ in (15) and is given by

$$v(\mathbf{x}, t_f) = \frac{1}{2}\mathbf{x}^\top S_f\mathbf{x}, \quad \text{for} \quad \mathbf{x} \in \mathcal{D}_\mathbf{x}. \quad (19)$$

Upon applying the principle of optimality to the optimal, expected performance index, (18, 15) and the chain rule for Markov stochastic processes in continuous time for the LQGP problem yields

$$\begin{aligned} 0 &= \frac{\partial v}{\partial t}(\mathbf{x}, t) + \underset{\mathbf{u}}{\text{Min}}\, [(A(t)\mathbf{x} + B(t)\mathbf{u} \\ &+ \mathbf{C}(t))^T\, \nabla_x[v](\mathbf{x}, t) + \frac{1}{2}\left(GG^T\right)(t) : \nabla_x[\nabla_x^\top[v]](\mathbf{x}, t) \\ &+ \frac{1}{2}\mathbf{x}^\top Q(t)\mathbf{x} + \frac{1}{2}\mathbf{u}^\top R(t)\mathbf{u} \\ &+ \sum_{k=1}^{q_1} \lambda_{1,k}(\mathbf{x}, t) \int_{\mathcal{Z}_{1,k}} [v(\mathbf{x} + [H_1(t)\cdot\mathbf{x}]_k z, t) \\ &- v(\mathbf{x}, t)]\, \phi_{1,k}(z, \mathbf{x}, t)dz \\ &+ \sum_{k=1}^{q_2} \lambda_{2,k}(\mathbf{x}, t) \int_{\mathcal{Z}_{2,k}} [v(\mathbf{x} + [H_2(t)\cdot\mathbf{u}]_k z, t) \\ &- v(\mathbf{x}, t)]\, \phi_{2,k}(z, \mathbf{x}, t), t)dz \\ &+ \sum_{k=1}^{q_3} \lambda_{3,k}(\mathbf{x}, t) \int_{\mathcal{Z}_{3,k}} [v(\mathbf{x} + \mathbf{H}_{3,k}(t)z, t) \\ &- v(\mathbf{x}, t)]\, \phi_{3,k}(z, \mathbf{x}, t)dz]\,, \quad (20) \end{aligned}$$

where the notation below defines the column arrays used in the Poisson terms,

$$\begin{aligned} [H_1(t)\cdot\mathbf{x}]_k &\equiv \left[\sum_{j=1}^{m} H_{1,i,k,j}(t)x_j\right]_{m\times 1}, \\ [H_2(t)\cdot\mathbf{u}]_k &\equiv \left[\sum_{j=1}^{n} H_{2,i,k,j}(t)u_j\right]_{m\times 1}, \\ \mathbf{H}_{3,k}(t) &\equiv [H_{3,i,k}(t)]_{m\times 1}, \quad (21) \end{aligned}$$

and where the double dot product is defined by

$$A : B = \sum_i \sum_j A_{i,j}B_{i,j} = \text{Trace}[AB^\top]. \quad (22)$$

The backward partial differential equation (PDE) (20) is known as the Hamilton-Jacobi-Bellman (HJB) equation and is subject to the final condition. The argument of the minimum is the optimal control, $\mathbf{u}^*(\mathbf{x}, t)$; if there are no control constraints the optimal control is known as the regular control, $\mathbf{u}_{\rm reg}(\mathbf{x},t)$.

To solve (20) subject to the final condition, for the LQGP problem a modification of the formal state decomposition of the solution for the usual LQG problem (for the usual LQG, see Bryson and Ho [3], Dorato et al. [4], or Lewis [9]) is assumed:

$$\begin{aligned} v(\mathbf{x},t) &= \frac{1}{2}\mathbf{x}^T S(t)\mathbf{x} + \mathbf{D}^T(t)\mathbf{x} + E(t) \\ &+ \frac{1}{2}\int_t^{t_f} \left(GG^T\right)(\tau) : S(\tau)d\tau. \end{aligned} \quad (23)$$

The final condition is satisfied, provided that

$$S(t_f) = S_f, \quad \mathbf{D}(t_f) = \mathbf{0}, \quad \text{and} \quad E(t_f) = 0. \quad (24)$$

The ansatz (23) would not, in general, be true for the state dependent case, but would be applicable if the Poisson noise is locally state independent, while globally state dependent. That is, the state domain is decomposed into subdomains, $\mathcal{D}_{\mathbf{x}} = \bigcup_i \mathcal{D}_{x_i}$, where the arrival rates and moments for all the Poisson processes are constant in the region $\mathcal{D}_{x_i}$ and can be expressed as

$$\left\{ \begin{array}{l} \Lambda(\mathbf{X}(t),t) = \Lambda_i(t) \\ \overline{\mathbf{Z}}(\mathbf{X}(t),t) = \overline{\mathbf{Z}}_i(t) \\ \sigma(\mathbf{X}(t),t) = \sigma_i(t) \end{array} \right\}, \quad \text{for} \quad \mathbf{X}(t) \in \mathcal{D}_{x_i}, \quad (25)$$

for all subdomains i. If there are any explicit dependence on $\mathbf{X}(t)$ then the resulting system would then form a LQGP/U problem (for more details see Westman and Hanson [14, 15, 16, 17]).

Assuming the ansatz (23) holds the regular, unconstrained optimal control, $\mathbf{u}^* = \mathbf{u}_{\rm reg}$, is given by

$$\mathbf{u}_{\rm reg}(t) = -\widehat{R}^{-1}(t)\widehat{B}^T(t)\left[S(t)\mathbf{x} + \mathbf{D}(t)\right]. \quad (26)$$

Assuming regular control, the coefficients for the optimal expected performance (23) are given by

$$\begin{aligned} 0_{m\times m} &= \dot{S}(t) + \left[A^T S + SA + Q\right](t) \\ &+ \widetilde{\Gamma}_1(t) - \left[S\widehat{B}\widehat{R}^{-1}\widehat{B}^T S\right](t), \end{aligned} \quad (27)$$

$$\begin{aligned} \mathbf{0}_{m\times 1} &= \dot{\mathbf{D}}(t) + \left[\left(A + (\Lambda_1\overline{\mathbf{Z}}_1)^T H_1^T\right)^T \mathbf{D}\right](t) \\ &+ \left[S\left(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3\right) - S\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t), \end{aligned} \quad (28)$$

$$\begin{aligned} 0 &= \dot{E}(t) + \left[\left(\mathbf{C} + H_3\Lambda_3\overline{\mathbf{Z}}_3\right)^T \mathbf{D}\right](t) \\ &+ \frac{1}{2}\left[\left(H_3^T S H_3\right) : \Lambda_3\overline{ZZ}_3 - \mathbf{D}^T\widehat{B}\widehat{R}^{-1}\widehat{B}^T\mathbf{D}\right](t), \end{aligned} \quad (29)$$

where

$$\begin{aligned} \Gamma_1(t) &\equiv \left[\left([H_1^T]_i S[H_1]_j : \Lambda_1\overline{ZZ}_1\right)(t)\right]_{m\times m} \\ &+ 2\left[\left(\Lambda_1\overline{\mathbf{Z}}_1\right)^T H_1^T S\right](t), \end{aligned} \quad (30)$$

$$\Gamma_2(t) \equiv \left[\left([H_2^T]_i S[H_2]_j : \Lambda_2\overline{ZZ}_2\right)(t)\right]_{n\times n}, \quad (31)$$

and

$$\begin{aligned} \overline{ZZ}_\ell(t) &\equiv \overline{\sigma}_\ell(t) + \left(\overline{\mathbf{Z}}_\ell\overline{\mathbf{Z}}_\ell^T\right)(t) \\ &= \left[\sigma_{\ell,i}\delta_{i,j} + \overline{Z}_{\ell,i}\overline{Z}_{\ell,j}\right]_{q_\ell\times q_\ell} \end{aligned} \quad (32)$$

for $\ell = 1$ to 3 with

$$\widehat{R}(t) \equiv R(t) + \widetilde{\Gamma}_2(t), \quad (33)$$

$$\widehat{B}(t) \equiv B(t) + ((\Lambda_2\overline{\mathbf{Z}}_2)^T H_2^T)(t), \quad (34)$$

and

$$\widetilde{\Gamma}_\ell \equiv (\Gamma_\ell + \Gamma_\ell^T). \quad (35)$$

Since the matrix R is positive definite, R^{-1} exists and then so does $\widehat{R}^{-1}$. Note (27) appears to have Riccati-like quadratic form, but in general is highly nonlinear through the S dependence of $\widehat{R}$ and if $H_\ell = [H_{\ell,i,j,k}]_{m\times q_\ell\times m_\ell}$, then $H_\ell^T = [H_{\ell,j,i,k}]_{q_\ell\times m\times m_\ell}$.

Due to uni-directional coupling of these matrix differential equations, it is assumed that the nonlinear matrix differential equation (27) for $S(t)$ is solved first and the result for $S(t)$ is substituted into equation (28) for $\mathbf{D}(t)$, which is then solved, and then both results for $S(t)$ and $\mathbf{D}(t)$ are substituted into equation (29) for the state-control independent term $E(t)$. Since $S(t)$ is a symmetric matrix by being defined with a quadratic form, only a triangle part of $S(t)$ need be solved, or $n\cdot(n+1)/2$ component equations. Thus, for the whole coefficient set $\{S(t), \mathbf{D}(t), E(t)\}$, only $n\cdot(n+1)/2 + n + 1$ component equations need to be solve, so that for large n the count is $\mathcal{O}(n^2/2)$, asymptotically, which is the same order of effort in getting the triangular part of $S(t)$.

3. LQGP Problem Formulation for MMS

Consider a MMS that produces the single consumable commodity. The MMS consists of k stages that form a linear sequence that is used to assemble the finished product. The mechanisms by which the input, *loading stage*, of raw materials and the delivery of finished products, *unloading stage*, are not considered as stages in the MMS. However, state dependent Poisson noises are used to model catastrophic events that affect the delivery of raw materials to the MMS. At time t in the manufacturing planning horizon for stage i, there are $n_i(t)$ operational workstations. For each stage i, all workstations are assumed to be identical and produce goods at the same rate $c_i(t)$ with a capacity of producing M_i parts per unit time. For each stage k of the manufacturing system the state of the MMS is given by the number of operational workstations, the surplus aggregate level, and the 3 state indicators for the effects of primary and secondary strikes and natural disasters on the MMS, respectively. Therefore the dimension of the state of the system is a $5k\times 1$ vector.

A primary strike is any strike that directly affects the assembly of the consumable good in such a way that when they occur the MMS is shut down and no goods are produced. A secondary strike is any strike that reduces the number of goods that can be produced, but does not disable the MMS. The impact of strikes and natural disasters on stage i, $s_{ij}(t)$, evolves according to the purely stochastic equation,

$$ds_{ij}(t) = -d\mathbf{P}_{ij}^S(s_{ij}(t),t) + d\mathbf{P}_{ij}^R(s_{ij}(t),t). \tag{36}$$

The term $-d\mathbf{P}_{ij}^S(s_{ij}(t),t)$ is used to model the effects of the start of an event and $d\mathbf{P}_{ij}^R(s_{ij}(t),t)$ is used to model the resolution of an event, where the events are $j=1$ primary strike, $j=2$ secondary strike, and $j=3$ natural disasters. The arrival rate for strikes is usually deterministic in the sense that normally there is a fixed date, say t_s, for the termination of a labor agreement, which if not resolved can lead to a strike. For all of these stochastic processes, the arrival rate is the mean time between the occurrences of such events and the amplitude or mark density function is modeled to represent the expected value and covariance for the event to occur. The values for state indicators are bounded by

$$0 \le s_{ij}(t) \le s_{ij}^{\max}, \tag{37}$$

where $s_{ij}^{\max} \le 1$ is the maximum impact the event can have on the MMS. The various events are considered to be additive and a functional is used to represent the net effect given by:

$$s_i(t) = \text{Min}[1, s_{i1}(t) + s_{i2}(t) + s_{i3}(t)]. \tag{38}$$

If $s_i(t)=0$, then there is no effect on the MMS. If $s_i(t)=1$ then no production takes place.

Each workstation is subject to failure and can be repaired. The mean time between failures and the repair duration is exponentially distributed. The evolution of the number of active workstations is bounded by

$$0 \le n_i(t) \le N_i, \tag{39}$$

for all time, process using *state dependent Poisson noises* given by

$$dn_i(t) = dP^R(n_i(t),t) - dP^F(n_i(t),t), \tag{40}$$

where $dP^R(n(t),t)$ and $dP^F(n(t),t)$ are used to model the repair and failure processes, respectively, which only depends on the current number of active workstations. This process forms a birth and death process or a random walk on the interval (39). The number of active workstations, $n_i(t)$, determines the arrival rates and mean mark amplitudes for failure and repair events respectively given by

$$1/\lambda_i^F = \left\{ \begin{array}{ll} 0, & n_i = 0 \\ 1/\lambda_i^F, & 1 \le n_i \le N_i \end{array} \right\}, \tag{41}$$

$$\overline{Z}_i^F = \left\{ \begin{array}{ll} 0, & n_i = 0 \\ \sum_{j=1}^{n_i} j\text{Pr}_j, & 1 \le n_i \le N_i \end{array} \right\}, \tag{42}$$

$$1/\lambda_i^R = \left\{ \begin{array}{ll} 1/\lambda_i^R, & 0 \le n_i < N_i \\ 0, & n_i = N_i \end{array} \right\}, \tag{43}$$

and

$$\overline{Z}_i^R = \left\{ \begin{array}{ll} \sum_{j=1}^{N_i - n_i} j\text{Pr}_{N_i - j}, & 0 \le n_i < N_i \\ 0, & n_i = N_i \end{array} \right\}, \tag{44}$$

with

$$\text{Pr}_i = \text{Pr}[n_i(t) = i] \tag{45}$$

which represents the probability of having i operational workstations at time t.

The surplus aggregate level represents the surplus (if positive) or shortfall (if negative) of the production of pieces that have successfully completed i stages of the manufacturing process is given by

$$\begin{aligned} da_i(t) &= [M_i c_i(t) n_i(t) + u_i(t) - d_i(t)]\, dt + g_i(t) dW_i(t) \\ &- \sum_{j=1}^{3} H_{ij}(t) d\mathbf{P}_{ij}^S(s_{ij}(t),t). \end{aligned} \tag{46}$$

The change in the surplus aggregate level, $da_i(t)$, is determined by the number of pieces that have successfully completed i stages of the manufacturing process (i.e., $M_i n_i(t) c_i(t) dt$), that are not defective, and are not consumed by stage $i+1$ (i.e., $d_i(t)dt$), and by the status of the workstations. The production rate $c_i(t)$ needs to be physically realizable with respect to the number of operational workstations and to the impact of strikes and natural disasters. The term $u_i(t)dt$ is used to adjust the production rate where the control $u_i(t)$ is expressed as the number of pieces per unit time. The term, $g_i(t)dW_i(t)$, is used to model the random fluctuations in the number of pieces produced, for example defective pieces. The demand term, $d_i(t)dt$, is the consumption of the pieces produced by stage i by stage $i+1$. The demand needs to be adjusted to compensate for over or under production as a result of strikes or natural disasters to meet the fixed production goal. The last term uses Poisson processes to represent the effects of strikes and natural disasters where the coefficients, $H_{ij}(t)$, are the expected value for the shortfall in the number of pieces produced.

The surplus aggregate level, $a_i(t)$, for stage i is dependent on the number of operational workstations, $n_i(t)$. The birth and death process for the number of operational workstations is an *embedded Markov chain* (see Taylor and Karlin [12], for instance), for the surplus aggregate level. Thus the birth and death process is used to describe the sojourn times for the discontinuous jumps in the surplus aggregate level due to the effects of workstation repair or failure. Hence, the surplus aggregate level is a piecewise continuous process whose discontinuous jumps are determined by the stochastic process for the number of operational workstations. Additionally, the processes for strikes and natural disasters also induce discrete large jumps in the surplus aggregate level through the sum in the last term of (46).

The demand rate $d_i(t)$ is the number of parts needed per unit time to insure that the manufacturing process is a continuous flow of work, so that the desired number of completed pieces are produced. The demand rate must also take into account, based on past history, a minimal buffer level sufficient to compensate for defective pieces as well as workstation failures, and to insure that the proper start-up surplus aggregate levels are present for the next planning horizon. In order for the MMS to be well posed, it is required that

$$0 \leq d_i(t) \leq M_i N_i \tag{47}$$

per unit time so that the production goal of the MMS is attainable.

In this formulation of the production scheduling, a rebalancing of the demand rate is used to compensate for the effects of strikes and natural disasters. Let PG denote the total production goal for the manufacturing planning horizon $T > 0$. The simple average demand rate which is used in this paper would initially be given as:

$$d_i(0) = \frac{PG}{T}, \tag{48}$$

for $i = 1$ to k.

Since the problem formulation presented here anticipates the strikes and natural disasters additional pieces are produced to compensate for these shortfalls in production. Here a rebalancing of demand should be done to adjust for these effects. For simplicity we focus only on the primary strike. A primary strike may occur at time t_s which coincides with the termination of a labor agreement. Assume that the resolution of the strike if one occurs is at time $t_{sr} = t_s + \delta_s$ where δ_s is the actual duration of the strike. Let $PP_i(t)$ be the cumulative number of nondefective pieces produced at stage i during the production interval $[0, t]$ for $t < T$. Depending on whether or not a strike occurs a rebalancing of the demand rate would be given by

$$d_i(t) = \frac{PG - PP_i(t)}{T - t}, \tag{49}$$

where

$$t = \left\{ \begin{array}{ll} t_s, & \text{No Strike} \\ t_{sr}, & \text{Strike} \end{array} \right\}. \tag{50}$$

This use of rebalancing of demand can also be used to consume pieces that remain in surplus due to workstation failure. Suppose a failure occurs at stage $i > 1$, then a surplus of pieces may accrue at stage i which would need to be consumed by stages i through k (the remaining stages of the MMS). The plant manager would need to decide the discipline for doing this. For example, the manager may choose to consume the pieces over the remaining manufacturing horizon as demonstrated above. Note, the production demand may be time dependent and therefore would require modifications to meet the objectives of the planning horizon, as would be the case of cyclic or seasonal demand of commodities.

The cost function used is the standard *time-to-go* or *cost-to-go* form (15, that is motivated by a *zero inventory* or *Just in Time* manufacturing discipline (see Hall [6] and Bielecki and Kumar [2]) while utilizing minimum control effort. In this formulation, the salvage cost, $S(t_f)$, is used to impose a penalty on surplus or shortfall of production at the end of the planning horizon. The term $Q(t)$ is used to penalize shortfall and surplus production during the planning horizon, this term is used to maintain a strict regimen on when the consumable goods are to be produced. The term $R(t)$ is used to enforce a minimum control effort penalty.

To solve this problem, assume the regular control (26) and solve the nonlinear system of ordinary differential equations (27,28,29). This allows the calculation of the production rates used by the plant manager of the MMS . The production rate, $c_i(t)$ is a utilization, that is the fraction of time busy. The physically realizable production rate is bounded by

$$0 \leq c_i(t) \leq c_i^{max}(t), \tag{51}$$

where

$$c_i^{max}(t) = \left\{ \begin{array}{ll} \widehat{s}_i(t), & i = 1 \\ \min[1, MPR_i], & 1 < i \leq k \end{array} \right\}, \tag{52}$$

with

$$MPR_i = \frac{\widehat{s}_{i-1}(t) c_{i-1}(t) n_{i-1}(t) M_{i-1}}{n_i(t) M_i}, \tag{53}$$

which is the unconstrained maximum physical production rate for stage i. The maximum production rate, $c_i^{max}(t)$, is the minimum value of the physical production rate, 1.00 or full utilization, and production limitations that arise due to a shortfall of production from the previous stage due to either machine failure, strikes, or natural disasters, where $\widehat{s}_i(t)$ is the total impact of strikes and natural disasters on stage i given by

$$\widehat{s}_i(t) = \mathrm{Max}\left[1 - \sum_{\mathrm{j}=1}^{3} \mathrm{s_{ij}(t)},\ 0\right]. \tag{54}$$

The strike and natural disaster influence is used to limit the amount of pieces that can be produced by a given stage and is bounded by $0 \leq \widehat{s}_i(t) \leq 1$, such that $\widehat{s}_i(t) = 0$ means that no production can occur.

In this formulation the production rate is a parameter of the dynamic system and is adjusted by the control decision. The regular controlled production level,

$$c_i^{\mathrm{reg}}(t) = \left\{ \begin{array}{ll} 0, & n_i(t) = 0 \\ c_i(t) + \frac{u_i^{\mathrm{reg}}(t)}{M_i n_i(t)}, & n_i(t) > 0 \end{array} \right\}, \tag{55}$$

which anticipates for the stochastic effects of workstation repair and failure, defective parts, strikes or natural disasters.

Note, that with the assumption of regular control, the surplus aggregate level will always be forced to be zero, therefore the regular controlled production level may not be physically realizable. In the case of a primary strike, $c_i(t)$=0 for all i, the regular controlled production level which is the same as the regular control would be the number of pieces that needs to be produced to force the surplus aggregate level to zero, which clearly is not physically realizable. The constrained controlled production level, $c_i^*(t)$, is the restriction of the regular controlled production level to be physically realizable and is given by

$$c_i^*(t) = \min[c_i^{\mathrm{reg}}(t), c_i^{max}(t)], \tag{56}$$

where $c_i^{max}(t)$ is given in (51). The constrained controlled production rate is used as the production rate for the workstations in the state equation for the surplus aggregate level (46).

4. Numerical Example of LQGP MMS

Here we present two path realizations for this model. Consider a MMS with $k = 3$ stages with a planning horizon of $T = 100$ days and a production goal of $PG = 57,500$ pieces which means the initial demand rate for all stages is given by $d_i(t) = 575$ pieces per day. Let the initial surplus aggregate level for all stages be zero, the total number of workstations, N_i, for each stage be 3, 5, and 4, respectively, the Gaussian random fluctuations of production is assumed absent ($g_i(t) = 0$ for $i = 1$ to 3), and that secondary strikes and natural disasters are not considered, for simplicity the strike impact state variable will be referred to as $s_i(t)$. The state variables the MMS are

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{a}(t) \\ \mathbf{s}(t) \end{bmatrix}_{9\times 1}, \tag{57}$$

where each component is a 3×1 vector. A single primary strike can occur at the beginning of day 63 (i.e., $t_s = 63$) of the planning horizon with an expected time of 14 days to resolve itself (i.e., $\mathrm{Mean}[\delta_s] = 14$), that is the arrival rates for the strike are given by

$$1/\lambda^S(s_i(t), t) = \left\{ \begin{array}{ll} (63 - t) \text{ days}, & t < 63 \\ 0 \text{ days}, & t \geq 63 \end{array} \right\}, \tag{58}$$

and

$$1/\lambda^R(s_i(t), t) = \left\{ \begin{array}{ll} 0 \text{ days}, & s_i(t) = 1 \\ 14 \text{ days}, & s_i(t) < 1 \end{array} \right\} \tag{59}$$

with an expected impact or shortfall of $575 * 14 = 8050$ pieces. The effects of a primary strike and its resolution on the MMS will disable or enable production for all stages.

The operational characteristics for the workstations are summarized in Table 4.

Stage i	Production Capacity, M_i (pieces/day)	Mean Time between Failure $1/\lambda_i^F$ (days)	Mean Time to Repair $1/\lambda_i^R$ (days)
1	238	85.0	2.50
2	143	75.0	1.50
3	178	90.0	1.75

Table 1: Operational workstation parameters.

Let $\Phi_{k,i,j}^R$ and $\Phi_{k,i,j}^F$ denote the discrete mark transition probabilities for the repair and failure, respectively, of $j-1$ workstations for stage k when there are i operational workstations, with transition matrices given by

$$\Phi_1^R = \begin{bmatrix} 0.00 & 0.95 & 0.05 \\ 0.00 & 1.00 & 0.00 \\ 1.00 & 0.00 & 0.00 \end{bmatrix}, \tag{60}$$

$$\Phi_1^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.90 & 0.10 \end{bmatrix}, \tag{61}$$

$$\Phi_2^R = \begin{bmatrix} 0.00 & 0.90 & 0.07 & 0.02 & 0.01 \\ 0.00 & 0.92 & 0.07 & 0.01 & 0.00 \\ 0.00 & 0.93 & 0.07 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}, \tag{62}$$

$$\Phi_2^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.95 & 0.05 & 0.00 & 0.00 \\ 0.00 & 0.94 & 0.05 & 0.01 & 0.00 \\ 0.00 & 0.92 & 0.05 & 0.02 & 0.01 \end{bmatrix}, \tag{63}$$

$$\Phi_3^R = \begin{bmatrix} 0.00 & 0.96 & 0.03 & 0.01 \\ 0.00 & 0.97 & 0.03 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}, \tag{64}$$

and

$$\Phi_3^F = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.95 & 0.05 & 0.00 \\ 0.00 & 0.90 & 0.07 & 0.03 \end{bmatrix}. \tag{65}$$

The cost functional used is (15) where the coefficient matrices are given by

$$S(t_f) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & S_f & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \tag{66}$$

$$S_f = \begin{bmatrix} 1.2 & 0.0 & 0.0 \\ 0.0 & 1.9 & 0.0 \\ 0.0 & 0.0 & 2.6 \end{bmatrix}, \tag{67}$$

$$Q(t) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & Q_2 & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \quad (68)$$

$$Q_2 = \begin{bmatrix} 0.9 & 0.0 & 0.0 \\ 0.0 & 1.6 & 0.0 \\ 0.0 & 0.0 & 2.3 \end{bmatrix}, \quad (69)$$

and

$$R(t) = \begin{bmatrix} 12000 & 0 & 0 \\ 0 & 12000 & 0 \\ 0 & 0 & 12000 \end{bmatrix}. \quad (70)$$

By comparing the coefficients of (1) with the state equations for the MMS (40,46,36) the deterministic coefficients are given by

$$A(t) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ \mathrm{diag}[\mathbf{M}]\mathrm{diag}[\mathbf{c}(t)] & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \quad (71)$$

$$B(t) = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \\ 0_{3\times3} \end{bmatrix}, \quad (72)$$

and

$$C(t) = \begin{bmatrix} \mathbf{0}_{3\times1} \\ -\mathbf{d}(t) \\ \mathbf{0}_{3\times1} \end{bmatrix}, \quad (73)$$

where $\mathrm{diag}[\mathbf{M}] = [M_i\delta_{i,j}]_{k\times k}$ is the diagonal matrix representation of the vector $\mathbf{M}$ and with the only *nonzero* stochastic process and corresponding coefficient matrix given by

$$d\mathbf{P}_3(\mathbf{X}(t), t) = \begin{bmatrix} d\mathbf{P}^R(\mathbf{n}(t), t) \\ d\mathbf{P}^F(\mathbf{n}(t), t) \\ dP^S(\mathbf{s}(t), t) \\ dP^R(\mathbf{s}(t), t) \end{bmatrix}, \quad (74)$$

and

$$H_3(t) = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8050 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8050 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8050 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \quad (75)$$

Using the above numerical values and assuming the regular control the temporal dependent coefficients $S(t)$, $d\mathbf{D}(t)$, and $E(t)$ can be determined from (27,28,29). With the temporal coefficients known the regular control can be determined from (26) for any state value. Finally, the regular control and value for the state can be used to determine the MMS operating parameters for the regular controlled production rate, $c_i^{\mathrm{reg}}(t)$, and constrained controlled production rate, $c_i^*(t)$.

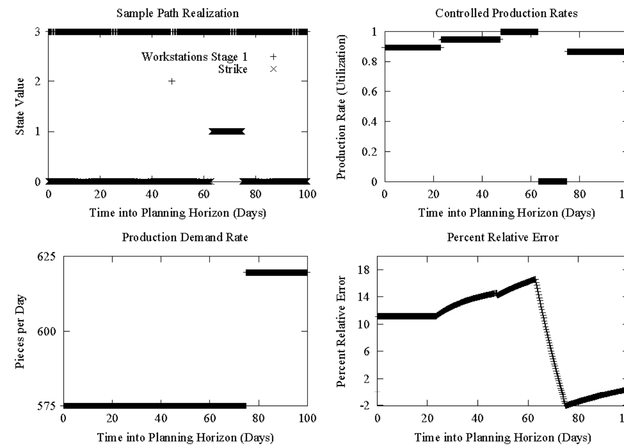

Figure 1: State sample path realization for active workstations, production rate for stage 1, demand rate, and percent relative error of throughput of stage 1.

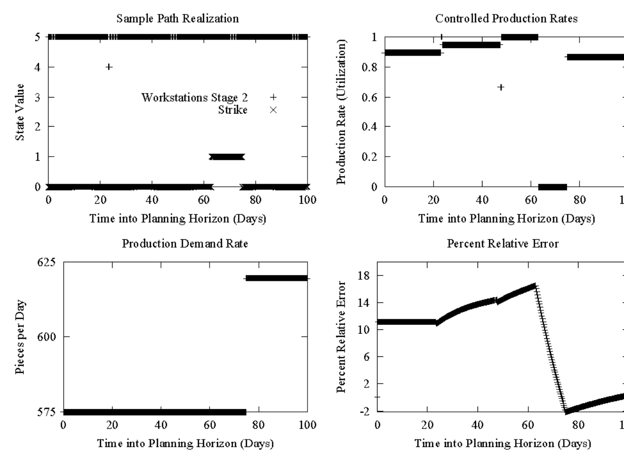

Figure 2: State sample path realization for active workstations, production rate for stage 2, demand rate, and percent relative error of throughput of stage 2.

Figures 1, 2, and 3 show the results for the case when a strike occurs with a rebalancing of the demand rate at the end of the strike which leads to a higher demand rate for the remaining manufacturing horizon. The percent relative error for this sample path for the final stage 3 which is the output of the MMS is 0.1406%. Figures 4, 5, and 6 are for the case when the strike does not occur and rebalancing of the demand rate occurs at $t_s = 63$ days which leads to a reduced demand rate for the remaining manufacturing horizon. The percent relative error for this sample path for the final stage 3 which is the output of the MMS is -0.006426%. The results presented here do not reflect the need to rebalance the demand rate for workstation repairs and failures.

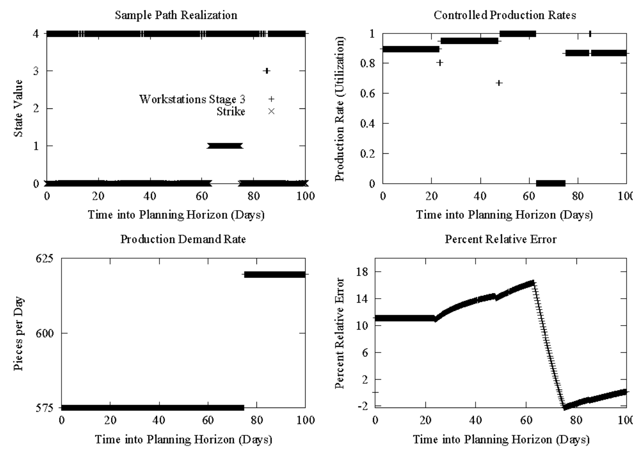

Figure 3: State sample path realization for active workstations, production rate for stage 3, demand rate, and percent relative error of throughput of stage 3.

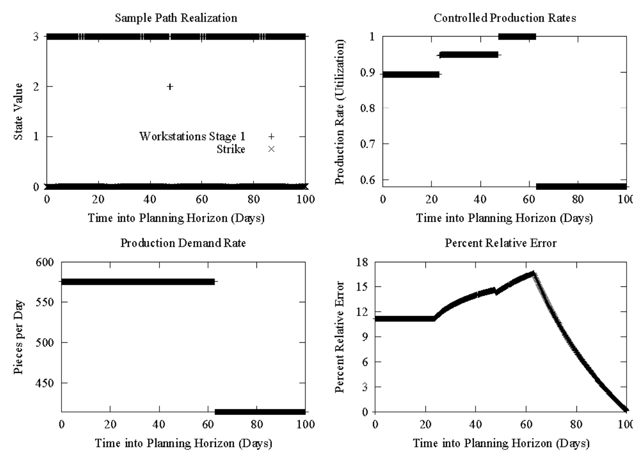

Figure 4: State sample path realization for active workstations, production rate for stage 1, demand rate, and percent relative error of throughput of stage 1.

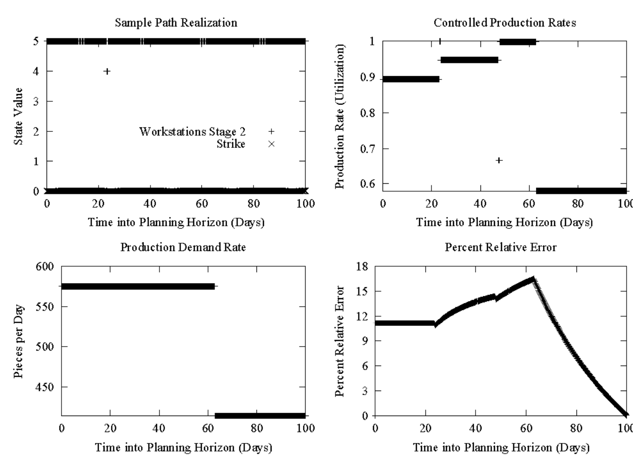

Figure 5: State sample path realization for active workstations, production rate for stage 2, demand rate, and percent relative error of throughput of stage 2.

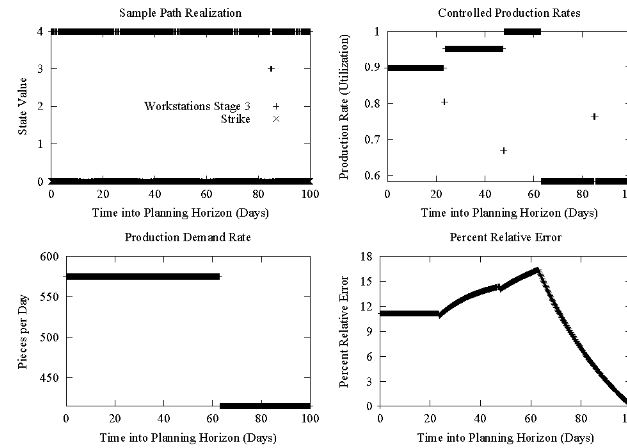

Figure 6: State sample path realization for active workstations, production rate for stage 3, demand rate, and percent relative error of throughput of stage 3.

5. Conclusions

A sudden labor strike or natural disaster can have catastrophic consequences that are much more serious than portrayed by the typical continuous state model, in addition to the jumps due to the random failure and repair of multistage manufacturing system (MMS) workstations. The model presented in this paper can be used to account for all of these random events, alter the demand rate to meet the production goal, and to determine the production rates of the workstations in order to minimize adverse financial effects. Our computational procedures lead to systematic approximations to the MMS model formulated here for strikes and other random catastrophic events.

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