

Abstract

The higher fixed point theorem for foliations. Applications to rigidity and integrality

We give applications of the higher Lefschetz theorems for foliations of [?], primarily involving Haefliger cohomology. These results show that the transverse structures of foliations carry important topological and geometric information. This is in the spirit of the passage from the Atiyah-Singer index theorem for a single compact manifold to their families index theorem, involving a compact fiber bundle over a compact base. For foliations, Haefliger cohomology plays the role that the cohomology of the base space plays in the families index theorem.

We obtain highly useful numerical invariants by pairing with closed holonomy invariant currents. In particular, we prove that the non-triviality of the higher \hat{A} genus of the foliation in Haefliger cohomology can be an obstruction to the existence of non-trivial leaf-preserving compact connected group actions. We then construct a large collection of examples for which no such actions exist. Finally, we relate our results to Connes' spectral triples, and prove useful integrality results.