79/80th Anniversary Midwest PDE Seminar

Sept. 14 - 17, 2017

All talks will take place in SEO 636

Thursday, Sept. 14

Time	**Speaker & affiliation**	**Title**
12:30--1:00	**Registration**	
1:00--1:50	Jared Wunsch (Northwestern)	Diffraction in spectral and scattering theory
2:00--2:50	Chanwoo Kim (U Wisconsin)	Vlasov-Poisson-Boltzmann system in bounded domains
3:00--3:25	Youngjoon Hong (UIC)	Numerical study of the 2^{nd} correct Hamiltonian model for unidirectional water waves
3:30--3:55	Dana Mendelson (U Chicago)	An infinite sequence of conserved quantities for the cubic Gross-Pitaevskii hierarchy on **R**
4:00--4:30	**Coffee break**	
4:30--4:55	Deniz Bilman (U Michigan)	A Robust Inverse Scattering Transform for the Focusing Nonlinear Schrödinger Equation
5:05--5:30	Jack Arbunich (UIC)	TBA

Friday, Sept. 15

Time	**Speaker & affiliation**	**Title**
9:00--9:50	Jeremy Marzuola (U North Carolina)	Degenerate 4^{th} order PDEs related to crystal surface relaxation:Microscopic to Macroscopic derivations, Analysis and Dynamics

10:00--10:30	**Coffee break**	
10:30--11:20	Natasha Pavlovic (UT Austin)	On well-posedness for Boltzmann's equation via dispersive tools
11:30--12:20	Yao Yao (Georgia Tech)	Long-time behavior of solutions to the 2D Keller-Segel equation with degenerate diffusion
12:30--2:00	**Lunch break**	
2:00--2:50	Kevin Zumbrun (Indiana U)	Stability of roll wave solutions in inclined shallow-water flow
3:00--3:50	Panagiotis Souganidis (U Chicago)	A theory for first- and second-order PDE on junctions (**Department Colloquium**)
4:00--4:50	**Department Tea**	
6:00	**Conference Dinner**	

Saturday, Sept. 16

Time	**Speaker & affiliation**	**Title**
9:00--9:50	Marian Bocea (Loyola)	TBA
10:00--10:30	**Coffee break**	
10:30--11:20	Arshak Petrosyan (Purdue)	TBA
11:30--12:20	Chongchun Zeng (Georgia Tech)	Instability, index theorems, and exponential dichotomy of Hamiltonian PDEs
12:30--2:00	**Lunch break**	
2:00--2:50	Burak Erdogan (UI Urbana-Champaign)	TBA
3:00--3:50	Serguei Denissov (UW Madison)	Wave operators for wave equations: some optimal results
4:00--4:30	**Coffee break**	

4:30--4:55	Trevor Leslie (UIC)	The Energy Measure for the Euler and Navier-Stokes Equations
5:05--5:30	Aseel Farhat (U Virginia)	TBA

Sunday, Sept. 17

Time	**Speaker & affiliation**	**Title**
9:00--9:50	Brian Seguin (Loyola)	Nonlocal curvature of surfaces and curves
10:00--10:30	**Coffee break**	
10:30--11:20	Mimi Dai (UIC)	Local well-posedness of the Hall-magneto-hydrodynamics system
11:30--11:55	Ebru Toprak (UIUC)	TBA
12:00--12:25	William Feldman (U Chicago)	Stability and Dynamic Stability of Serrin's Problem
12:30--12:55	Jayin Jin (Georgia Tech)	Local dynamics near solitary waves of the supercritical gKDV equations.

Abstracts

D. Bilman: We propose a modification of the standard inverse scattering transform for the focusing nonlinear Schrödinger equation (also other equations by natural generalization). The purpose is to deal with arbitrary-order poles and potentially severe spectral singularities in a simple and unified way. As an application, we use the modified transform to place the Peregrine breather solution and related `rogue wave' solutions in an inverse-scattering context for the first time. This allows one to directly study the stability of such solutions. The modified transform method also allows rogue waves to be generated on top of other structures by elementary Darboux transformations, rather than the generalized Darboux transformations in the literature.

M. Dai: We will talk about the 3D Hall-magneto-hydrodynamics system, due to the presence of the Hall term, which is different from the magneto-hydrodynamics (MHD). The Hall term destroys the scaling of the usual MHD. We will discuss the local well-posedness of the Hall-MHD in optimal Sobolev spaces.

W. Feldman: I will explain a new result about the quantitative stability of Serrin's symmetry problem and its connection to a dynamic model for the contact angle driven motion of capillary drops.

C. Kim: We construct a unique global solution of the Vlasov-Poisson-Boltzmann system in bounded domains.

S. Denissov: We will consider the wave equation in three dimensions in the case when potential decays slowly. The proof for the existence of wave operators will be explained.

Y. Hong: Attention is given to the study of the propagation of long-crested wave motions with an incompressible perfect fluid in a uniform horizontal channel. When the fluid motion is irrotational, inviscid and uniform in the cross channel direction, the two-dimensional version of the Euler equations provide a good model of waves on the surface of water. However, in many practical applications, full water wave model appears to be more complex than is necessary, and consequently further approximations are often made in the shallow water regime. In this talk, we numerically study the unidirectional fifth-order (second-order correct) KdV-BBM, and examine behavior of solutions arising from long-crested waves propagation. A numerical algorithm based on the Fourier spectral method is presented, and their numerical convergence is tested. A clean solitary wave is generated numerically, and we report a sequence of numerical tests on the validation of solitary wave approximation. Utilizing the generated clean solitary waves, various numerical experiments on numerical stability, interaction, and resolution of solitary waves are performed. A comparison is made between the KdV-BBM and the fifth-order equations using scaled and unscaled models.

J. Jin: We classify the local dynamics near the solitary waves of the supercritical gKDV equations. We proved that there exists a co-dimensional two center manifold, such that if the initial data is not on the center manifold, then the flow will exit a neighborhood of the solitary waves exponentially fast either in positive time or in negative time. Moreover, we show the orbital stability of the solitons on the center manifold, which also implies the global existence of the solutions on the center manifold and the local uniqueness of the center manifolds. Furthermore, applying a theorem of Martel and Merle, we have that the solitons are asymptotically stable on the center manifold in some local sense. This is a joint work with Zhiwu Lin and Chongchun Zeng.

T. Leslie: The potential failure of energy equality for a solution *u* of the Euler or Navier-Stokes equations can be quantified using a so-called `energy measure': the weak-* limit of the measures $|u(t)|^2dx$ as *t* approaches the first possible blowup time. We show that membership of *u* in certain (weak or strong) $L^q L^p$ classes implies uniform boundedness of a certain upper s-density of *E*, giving a uniform lower bound on the lower local dimension of *E*. We also define and give lower bounds on the `concentration dimension' associated to *E*, which is the Hausdorff dimension of the smallest set on which energy can concentrate. Both the lower local dimension and the concentration dimension of *E* measure the departure from energy equality. As an application of our results, we prove that any solution to the 3-dimensional Navier-Stokes Equations which is Type-I in time must satisfy the energy equality at the first blowup time. Furthermore, we give new criteria for energy conservation (equality) in terms of the dimension of the singularity set and classical $L^q L^p$ conditions. This is joint work with Roman Shvydkoy.

J. Marzuola: We will discuss the derivation and analysis of a family of 4th order nonlinear PDEs that arise in the study of the evolution of crystal surfaces due to thermodynamic fluctuations. This will include discussions of various joint works with Jon Weare, Jianfeng Lu, Dio Margetis, Jian-Guo Liu and Anya Katsevich.

D. Mendelson: We consider the (de)focusing cubic Gross-Pitaevskii (GP) hierarchy on R, which is an infinite hierarchy of coupled linear non-homogeneous PDE which appears in the derivation of the cubic nonlinear Schrodinger (NLS) equation from quantum many-particle systems. Motivated by the fact that the cubic NLS on **R** is an integrable equation which admits infinitely many conserved quantities, we exhibit an infinite sequence of operators which generate analogous conserved quantities for the GP hierarchy. This is joint work with Andrea Nahmod, Natasa Pavlovic, and Gigliola Staffilani.

N. Pavlovic: Boltzmann's equation is an evolutionary partial differential equation which describes the behavior of a dilute gas of identical particles in a specific scaling limit. In this talk we will focus on the theory of local well-posedness. Our main intention is not to investigate optimal regularity spaces for solving Boltzmann equation. Rather, we demonstrate the close connection between the Boltzmann equation and nonlinear Schrödinger equations in the density matrix formulation; this connection has been recognized implicitly for some time, but we wish to make it quite explicit and to the best of our knowledge this is the first time such an explicit connection has been established.

B. Seguin: Motivated by generalizations of the Ginzburg-Landau energy and the diffusion equation in which derivatives are replaced by fractional derivatives, Caffarelli, Roquejoffre, and Savin studied the minimizers of a fractional perimeter functional on sets involving a parameter between 0 and 1. Such minimizers have to satisfy a pointwise condition on their boundary, which can be used to define a notion of nonlocal mean-curvature. This definition only holds for surfaces which are the boundary of a set. I will describe how to define a nonlocal notion of mean-curvature for any surface by introducing a fractional area functional and considering its minimizers. Moreover, I will describe how these ideas can be extended to curves by defining a fractional length and an associated nonlocal curvature for a curve.

P. Souganidis: I will present recent developments about first- and second-order PDE on junctions. I will discuss some concrete applications, the well-posedness of the problems as well as stability of asymptotic limits including fattening of domains. If time permits, I will also present some new results about scalar conservation laws along the same spirit. This is joint work with Pierre-Louis Lions.

J. Wunsch: The diffraction of waves is a phenomenon whose study dates back to at least the 17th century, but remains a challenge for rigorous analysis. I will discuss some recent work on the phenomenon of diffraction by conic singularities, and its application to the analysis of the spectrum of the Laplace operator and the distribution of resonances in scattering theory. We expect that an acoustic wave striking the exterior of a polygon should cause prolonged ringing (\`resonance') which would not be present for a smooth obstacle, and I will present some results to substantiate this claim.

Y. Yao: The Keller-Segel equation is a nonlocal PDE modeling the collective motion of cells attracted by a self-emitted chemical substance. When this equation is set up in 2D with a degenerate diffusion term, it is known that solutions exist globally in time, but their long-time behavior remains unclear. In a joint work with J.A.Carrillo, S.Hittmeir and B.Volzone, we prove that all stationary solutions must be radially symmetric up to a translation, and use this to show convergence towards the stationary solution as the time goes to infinity. I will also discuss another joint work with K.Craig and I.Kim, where we let the power of degenerate diffusion go to infinity in the 2D Keller-Segel equation. Using a combination of viscosity solution theory and gradient flow approach, we will show that if the initial data is a characteristic function, the solution solves a free boundary problem similar to the Hele-Shaw equation, and it will converge to the characteristic function of a disk as the time goes to infinity with certain convergence rate.

C. Zeng: Motivated by the stability/instability analysis of coherent states (standing waves, traveling waves, etc.) in nonlinear Hamiltonian PDEs such as BBM, GP, and 2-D Euler equations, we consider a general linear Hamiltonian system $u_t = JLu$ in a real Hilbert space **X**, the energy space. The main assumption is that the energy functional $\langle Lu, u\rangle$ has only finitely many negative dimensions: $n^{-L} < \infty$. Our first result is an L-orthogonal decomposition of **X** into closed subspaces so that JL has a nice structure. Consequently, we obtain an index theorem which relates $n^{-(L)}$ and the dimensions of subspaces of generalized eigenvectors of some eigenvalues of JL, along with some information on such subspaces. Our third result is the linear exponential trichotomy of the group $e^{t\,JL}$. This includes the nonexistence of exponential growth in the finite co-dimensional invariant center subspace and the optimal bounds on the algebraic growth rate there. Next we consider the robustness of the stability/instability under small Hamiltonian perturbations. In particular, we give a necessary and sufficient condition on whether a purely imaginary eigenvalues may become hyperbolic under small perturbations. Finally we revisit some nonlinear Hamiltonian PDEs. This is a joint work with Zhiwu Lin.

K. Zumbrun: We review recent developments in stability of periodic roll-wave solutions of the Saint Venant equations for inclined shallow-water flow. Such waves are well-known instances of hydrodynamic instability, playing an important role in hydraulic engineering, for example, flow in a channel or dam spillway. Until recently, the analysis of their stability has been mainly by formal analysis in the weakly unstable or `near-onset' regime. However, hydraulic engineering applications are mainly in the strongly unstable regime far from onset. We discuss here a unified framework developed together with Blake Barker, Mat Johnson, Pascal Noble, Miguel Rodrigues, and Zhao Yang for the study of roll wave stability across all parameter regimes, by a combination of rigorous analysis and numerical computation. The culmination of our analysis is a complete stability diagram, of which the low-frequency stability boundary is, remarkably, given explicitly as the solution of a cubic equation in the parameters of the solution space.