

Solving a quadratic equation

a case study

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Outline

The Problem

Picturing the
Solution

Some Algebra

The Formula

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A pesky problem

Your paycheck has been held up, and they keep asking,
“Are you really a mathematician?”

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What to do?

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And then the idea hits you - you'll show them you can solve
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If that doesn't convince the admin type, what will?

Choosing a quadratic equation

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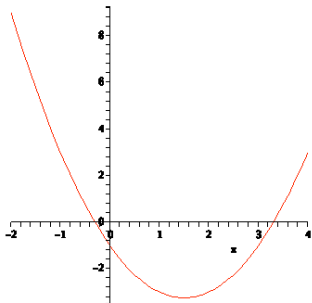
$$y = x^2 - 3x - 1$$

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“Give the solution to 10 decimals, and we’ll show you the money!”

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“Give the solution to 10 decimals, and we’ll show you the money!”

“Oh, for @#%& sake!”

factor, factor, complete...

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$$0 = (x - 3/2)^2 - 13/4$$

Progress

Now let's solve it:

$$0 = (x - 3/2)^2 - 9/4 - 4/4 \implies (x - 3/2)^2 = 13/4$$

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$$\begin{aligned}0 &= (x - 3/2)^2 - 9/4 - 4/4 &\implies (x - 3/2)^2 &= 13/4 \\ & &\implies (x - 3/2) &= \pm\sqrt{13/4}\end{aligned}$$

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$$x = 3/2 + \sqrt{13/4}, \text{ or}$$

$$x = 3.30277563773199464655961063373524797312564828692262310635522$$

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There are two solutions:

$$x = 3/2 + \sqrt{13/4}, \text{ or}$$

$$x = 3.30277563773199464655961063373524797312564828692262310635522$$

and $x = 3/2 - \sqrt{13/4}$, or

$$x = -0.302775637731994646559610633735247973125648286922623106355$$

Mathematical Proof

Solving a
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Give them the Magic Formula,

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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and tell them to try this first next time...