

# Solving a quadratic equation

## a case study

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The Problem

Picturing the Solution

Some Algebra

The Formula

## A pesky problem

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If that doesn't convince the admin type, what will?

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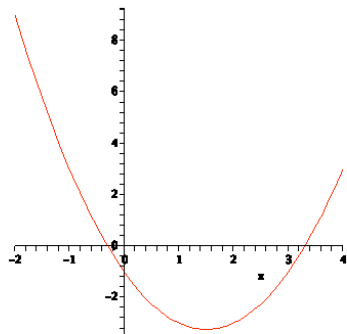
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“Oh, for @#%& sake!”

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$$0 = (x - 3/2)^2 - 13/4$$



## Progress

Now let's solve it:

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and  $x = 3/2 - \sqrt{13/4}, \text{ or}$

$$x = -0.30277563773199464655961063373524797312564828692262310635$$



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and tell them to try this first next time...