Solving a quadratic equation

a case study

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Math 589 Presentation – October 30, 2007

A pesky problem

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If that doesn't convince the admin type, what will?

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3. $x^2 - 3x - 1 = 0$ (sort of fancy... just right!)

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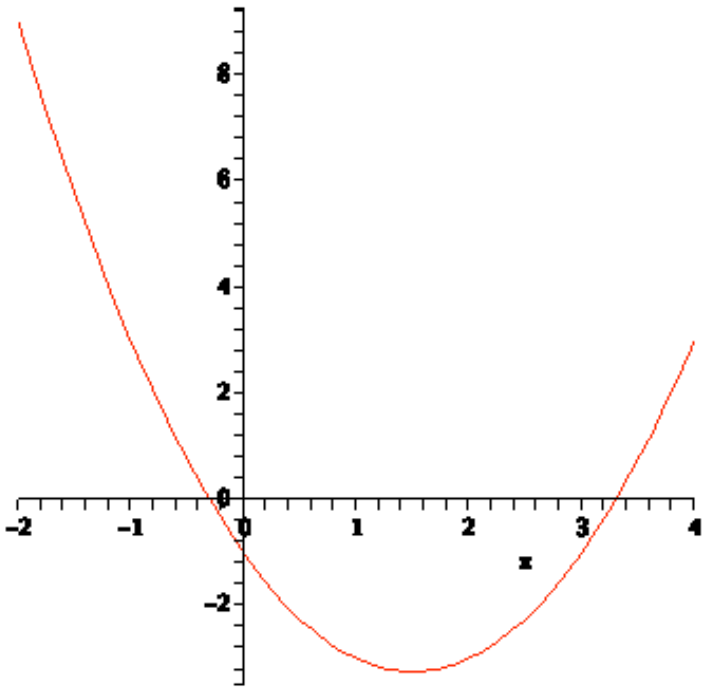

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"Oh, for @#%& sake!"

factor, factor, complete...

$$0 = x^2 - 3x - 1$$

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Progress

Now let's solve it:

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$x = 3.302775637731994646559610633735247973125648286922623106350$

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$x = 3/2 + \sqrt{13/4}$, or

$x = 3.302775637731994646559610633735247973125648286922623106 35$

and $x = 3/2 - \sqrt{13/4}$, or

$x = -0.302775637731994646559610633735247973125648286922623106 3$

Mathematical Proof

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$$ax^2 + bx + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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and tell them to try this first next time...