Solving a quadratic equation

a case study

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The Problem

Picturing the Solution

Some Algebra

The Formula
A pesky problem

Your paycheck has been held up, and they keep asking,

“Are you really a mathematician?”
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How to convince them?
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What to do?
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What to do?

And then the idea hits you - you’ll show them you can solve a quadratic equation!
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“Are you really a mathematician?”

How to convince them?
What to do?
And then the idea hits you - you’ll show them you can solve a quadratic equation!
If that doesn’t convince the admin type, what will?
Choosing a quadratic equation

Now, it is only a matter to select a quadratic equation which will impress them.

1. $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)
2. $x^2 - 2x + 1 = 0$ (more of the same)
3. $x^2 - 3x - 1 = 0$ (sort of fancy... just right!)
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Grab your calculators:

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Let’s try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$
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Not even close...

“You want money for your one lousy graph?”
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“Give the solution to 10 decimals, and we’ll show you the money!”
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“You want money for your one lousy graph?”

“Give the solution to 10 decimals, and we’ll show you the money!”

“Oh, for @#$%& sake!”
factor, factor, complete...

\[ 0 = x^2 - 3x - 1 \]
factor, factor, complete...

\[ 0 = x^2 - 3x - 1 \]
\[ 0 = x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1 \]
factor, factor, complete...

\[
0 = x^2 - 3x - 1
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0 = x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1
\]

\[
0 = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{4}{4}
\]
factor, factor, complete...

\[0 = x^2 - 3x - 1\]
\[0 = x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1\]
\[0 = (x - 3/2)^2 - 9/4 - 4/4\]
\[0 = (x - 3/2)^2 - 13/4\]
Now let’s solve it:

\[0 = (x - \frac{3}{2})^2 - \frac{9}{4} - \frac{4}{4} \implies (x - \frac{3}{2})^2 = \frac{13}{4}\]
Now let’s solve it:

\[ 0 = (x - 3/2)^2 - 9/4 - 4/4 \implies (x - 3/2)^2 = 13/4 \implies (x - 3/2) = \pm \sqrt{13/4} \]
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Think this is enough to get the money?
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\[ \implies (x - 3/2) = \pm \sqrt{13/4} \]
\[ \implies x = 3/2 \pm \sqrt{13}/4 \]

Think this is enough to get the money?

Not likely...
Pay Up!

There are two solutions:

\[ x = \frac{3}{2} + \sqrt{\frac{13}{4}}, \text{ or } x = \frac{3}{2} - \sqrt{\frac{13}{4}}. \]
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\[ x = \frac{3}{2} + \sqrt{\frac{13}{4}}, \text{ or} \]

\[ x = 3.30277563773199464655961063373524797312564828692262310635 \]
Pay Up!

There are two solutions:

\[ x = \frac{3}{2} + \sqrt{\frac{13}{4}}, \text{ or } \]
\[ x = 3.30277563773199464655961063373524797312564828692262310635\ldots \]

and \[ x = \frac{3}{2} - \sqrt{\frac{13}{4}}, \text{ or } \]
\[ x = -0.3027756377319946465596106337352479731256482869226231063\ldots \]
Mathematical Proof

The final proof that we are Mathematicians?
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Give them the Magic Formula,

\[ ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Mathematical Proof

The final proof that we are Mathematicians?

Give them the Magic Formula,

\[ ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

and tell them to try this first next time...