Solving a quadratic equation

a case study

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1. The Problem

2. Picturing the Solution

3. Some Algebra

4. The Formula
A pesky problem

Your paycheck has been held up, and they keep asking, “Are you really a mathematician?”
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How to convince them?
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And then the idea hits you - you’ll show them you can solve a quadratic equation!
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What to do?

And then the idea hits you - you’ll show them you can solve a quadratic equation!

If that doesn’t convince the admin type, what will?
Choosing a quadratic equation

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3. $x^2 - 3x - 1 = 0$
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2. \( x^2 - 2x + 1 = 0 \) (more of the same)
3. \( x^2 - 3x - 1 = 0 \) (sort of fancy... just right!)
A picture may be worth a thousand words, but is it worth a thousand bucks?
Grab your calculators:

A picture may be worth a thousand words, but is it worth a thousand bucks?

Let’s try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$
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Let’s try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$
Not even close...

“You want money for your one lousy graph?”
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“You want money for your one lousy graph?”

“Give the solution to 10 decimals, and we’ll show you the money!”
Not even close...

“‘You want money for your one lousy graph?’”

“‘Give the solution to 10 decimals, and we’ll show you the money!’”

“‘Oh, for @#%& sake!’”
factor, factor, complete...

0 = x^2 - 3x - 1
factor, factor, complete...

\[ 0 = x^2 - 3x - 1 \]

\[ 0 = x^2 - 3x + (\frac{-3}{2})^2 - (\frac{3}{2})^2 - 1 \]
factor, factor, complete...

\[ 0 = x^2 - 3x - 1 \]
\[ 0 = x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1 \]
\[ 0 = (x - \frac{3}{2})^2 - \frac{9}{4} - \frac{4}{4} \]
factor, factor, complete...

\[ 0 = x^2 - 3x - 1 \]
\[ 0 = x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1 \]
\[ 0 = (x - \frac{3}{2})^2 - \frac{9}{4} - \frac{4}{4} \]
\[ 0 = (x - \frac{3}{2})^2 - \frac{13}{4} \]
Progress

Now let’s solve it:

\[ 0 = (x - 3/2)^2 - 9/4 - 4/4 \implies (x - 3/2)^2 = 13/4 \]
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$$\implies (x - 3/2) = \pm \sqrt{13/4}$$
Now let’s solve it:

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Think this is enough to get the money?
Now let’s solve it:

\[ 0 = (x - 3/2)^2 - 9/4 - 4/4 \quad \Rightarrow \quad (x - 3/2)^2 = 13/4 \]

\[ \Rightarrow \quad (x - 3/2) = \pm \sqrt{13/4} \]

\[ \Rightarrow \quad x = 3/2 \pm \sqrt{13}/4 \]

Think this is enough to get the money?

Not likely...
Pay Up!

There are two solutions:
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\[ x = \frac{3}{2} + \sqrt{\frac{13}{4}}, \text{ or } \]

\[ x = 3.302775637731994646559610633735247973125648286922623474146505222602309541009245359... \]
Pay Up!

There are two solutions:

\[ x = \frac{3}{2} + \sqrt{\frac{13}{4}}, \text{ or } \]
\[ x = 3.302775637731994646559610633735247973125648286922623 \]

and \( x = \frac{3}{2} - \sqrt{\frac{13}{4}}, \text{ or } \)

\[ x = -0.302775637731994646559610633735247973125648286922623 \]
The final proof that we are Mathematicians?
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Give them the Magic Formula,

\[ ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
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\[ ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

and tell them to try this first next time...