

Dynamics and spectral geometry of foliations: references *

Lectures by Steven Hurder[†]

1. **January 31:** *Foliations and groupoids*. Here are some papers on foliations as groupoids and their classifying spaces.
 - (a) *Sur certaines propriétés topologiques des variétés feuilletées* by G. Reeb, **Act. Sci. et Ind.** 1183:91–154, 1952.
 - (b) *Structures feuilletées et cohomologie à valeur dans un faisceau de groupoïdes* by A. Haefliger, **Comment. Math. Helv.**, 32:248–329, 1958.
 - (c) *Variétés feuilletées* by A. Haefliger, **Ann. Scuola Norm. Sup. Pisa**, 16:367–397, 1962.
 - (d) *Foliations and foliated vector bundles* by J. Milnor, mimeographed notes, MIT, 1970. Available at http://www.foliations.org/surveys/FoliationLectNotes_Milnor.pdf
 - (e) *Feuilletages sur les variétés ouvertes* by A. Haefliger, **Topology**, 9:183–194, 1970.
 - (f) *Homotopy and integrability* by A. Haefliger, in **Manifolds—Amsterdam 1970**, Lect. Notes in Math. Vol. 197, Springer–Verlag, 1971:133–163.
 - (g) *On the de Rham theory of certain classifying spaces* by R. Bott, H. Shulman, and J. Stasheff, **Advances in Math.**, 20:43–56 1976.
 - (h) *Continuous cohomology of spaces with two topologies* by M. Mostow, **Mem. Amer. Math. Soc.**, Vol. 7, 1976.
 - (i) *Foliations and monoids of embeddings* by D. McDuff, in Geometric Topology (Proc. Georgia Topology Conf., Athens, Ga., 1977), Academic Press, New York, 1979:429–444.
 - (j) *Dual homotopy invariants of G -foliations* by S. Hurder, **Topology**, 20(4):365–387, 1981.
 - (k) *K -Theory and C^* -algebras of Lie groups and foliations* by M. Penington, **D.Phil Thesis**, Oxford University, 1983.
 - (l) *Groupoïdes d’holonomie et classifiants* by A. Haefliger, in **Transversal structure of foliations (Toulouse, 1982)**, Asterisque, 177-178, Société Mathématique de France, 1984:70–97.
 - (m) *The classifying space of smooth foliations* by S. Hurder, **Illinois J. Math.**, 29:108–133, 1985.
 - (n) *Every surface is a leaf* by J. Cantwell and L. Conlon, **Topology**, 26:265–285, 1987.
 - (o) *On the foliated products of class C^1* , by T. Tsuboi, **Ann. of Math.**, 130:227–271, 1989.
 - (p) *Classifying toposes and foliations* by I. Moerdijk, **Ann. Inst. Fourier (Grenoble)**, 41:189–209, 1991.
 - (q) *Models for the leaf space of a foliation* by I. Moerdijk, **European Congress of Mathematics, Vol. I (Barcelona, 2000)**, Progr. Math. 201:481–489, 2001.
 - (r) *Groupoids and foliations* by A. Haefliger, in Groupoids in analysis, geometry, and physics (Boulder, CO, 1999), Contemp. Math., Vol. 282:83–100, 2001.
 - (s) *Foliations and compactly generated pseudogroups* by A. Haefliger, in Foliations: geometry and dynamics (Warsaw, 2000), World Sci. Publ., pages 275–295, 2002.
 - (t) *Naissance des feuilletages, d’Ehresmann-Reeb à Novikov* by A. Haefliger, preprint 2003. Available at <http://foliations.org/surveys/Feuilletages.pdf>

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2. **February 2:** “Stackie” dynamics. Here are some papers which discuss dynamical properties of a foliation which are well-defined up to Morita equivalence of the associated groupoid.

- (a) *Foliations with measure-preserving holonomy* by J. Plante, **Ann. of Math.**, 102:327–361, 1975.
- (b) *Measure preserving pseudogroups and a theorem of Sacksteder* by J. Plante, **Ann. Inst. Fourier (Grenoble)**, 25:237–249, 1975.
- (c) *Polynomial growth in holonomy groups of foliations* by J. Plante and W. Thurston, **Comment. Math. Helv.**, 51:567–584, 1976.
- (d) *Cycles for the dynamical study of foliated manifolds and complex manifolds* by D. Sullivan, **Invent. Math.**, 36:225–255, 1976.
- (e) *Leaves whose growth is neither exponential nor polynomial* by G. Hector, **Topology**, 16:451–459, 1977.
- (f) *Growth of leaves* by J. Cantwell and L. Conlon, **Comment. Math. Helv.**, 53:93–111, 1978.
- (g) *Holonomy and averaging in foliated sets* by S. Goodman and J. Plante, **J. Differential Geom.**, 14:401–407, 1979.
- (h) *Ergodic theory and Weil measures for foliations* by S. Hurder and A. Katok, **Annals of Math**, 126:221–275, 1987.
- (i) *Entropie géométrique des feuilletages* by É. Ghys, R. Langevin, and P. Walczak, **Acta Math.**, 160:105–142, 1988.
- (j) *Exceptional minimal sets of $C^{1+\alpha}$ actions on the circle* by S. Hurder, **Ergodic Theory and Dynamical Systems**, 11:455–467, 1991.
- (k) *Coarse cohomology and index theory on complete Riemannian manifolds* by J. Roe, **Mem. Amer. Math. Soc.** Vol. 104, 1993.
- (l) *Exotic index theory for foliations* by S. Hurder, *preprint*, 1993.
- (m) *Coarse geometry of foliations* by S. Hurder, **Geometric Study of Foliations, Tokyo 1993**, (eds. Mizutani et al). World Scientific, 1995: 35–96.
- (n) *Exotic index theory and the Novikov conjecture* by S. Hurder, in *Novikov conjectures, index theorems and rigidity, Vol. 2 (Oberwolfach, 1993)*, Vol. 227 of *London Math. Soc. Lecture Note Ser.*, pages 255–276. Cambridge Univ. Press, 1995.
- (o) *Manifolds which cannot be leaves of foliations* by O. Attie and S. Hurder, *Topology*, 35(2):335–353, 1996.
- (p) *Amenability, hyperfiniteness, and isoperimetric inequalities* by V. Kaimanovich, **C. R. Acad. Sci. Paris Sér. I Math.**, 325:999–1004, 1997.
- (q) *Amenable groupoids* by C. Anantharaman-Delaroche and J. Renault, **Monographies de L’Enseignement Mathématique**, Vol. 36, 2000.
- (r) *Equivalence relations with amenable leaves need not be amenable* by V. Kaimanovich, in **Topology, ergodic theory, real algebraic geometry**, Amer. Math. Soc. Transl. Ser. 2 Vol. 202:151–166, 2001.
- (s) **Lectures on coarse geometry** by J. Roe, University Lecture Series, Vol. 31, Amer. Math. Soc., 2003.
- (t) *What is ... a coarse space?* by J. Roe, **Notices Amer. Math. Soc.**, 53:668–669, 2006.

3. **February 7: Analysis of leafwise operators.** This is a selection of papers concerning the basic properties of analysis along the leaves of a foliation, considered as a complete Riemannian immersed submanifold. A few selected papers about groupoid C^* -algebras are also included.
- (a) *Harmonic integrals on foliated manifolds* by B.L. Reinhart, **American Jour. Math.**, 81:529–536, 1959.
 - (b) *Essential self-adjointness of powers of generators of hyperbolic equations* by P. Chernoff, **J. Functional Analysis**, 12: 401–414, 1973.
 - (c) *A groupoid approach to C^* -algebras* by J. Renault, **Lecture Notes in Math. vol 793** Springer-Verlag, 1980.
 - (d) *Sur les représentations et idéaux de la C^* -algèbre d'un feuilletage* by T. Fack and G. Skandalis, **Jour. Operator Theory**, 8:95–129, 1982.
 - (e) *Some Riemannian and dynamical invariants of foliations* by R. Brooks, in **Differential geometry (College Park, Md., 1981/1982)**, Progress in Mathematics, Vol. 32, Birkhäuser, Boston, MA, 1983:56-72.
 - (f) *The spectral geometry of foliations* by R. Brooks, **Amer. J. Math.**, 106:1001–1012, 1984.
 - (g) *A theorem on coincidence of spectra for tangentially elliptic operators on foliated manifolds* by Yu. A. Kordyukov, **Funct. Anal. Appl.**, 19:327–328, 1985.
 - (h) *On the coincidence of spectra of random elliptic operators* by S.M. Kozlov and M.A. Šubin, **Math. USSR Sbornik**, 51:455–471, 1985.
 - (i) *Finite propagation speed and Connes' foliation algebra* by J. Roe, **Math. Proc. Camb. Phil. Soc.**, 102:459–466, 1987.
 - (j) *An index theorem on open manifolds, I,II* by J. Roe, **Jour. Differential Geom.**, 27:87–113, 115–136, 1988.
 - (k) *Remark on a paper of É. Ghys* by J. Roe, **J. Funct. Anal.**, 89:150–153, 1990.
 - (l) *The heat equation for Riemannian foliations* by S. Nishikawa, M. Ramachandran, and Ph. Tondeur, **Trans. Amer. Math. Soc.**, 319:619–630, 1990.
 - (m) *Von Neumann spectra near zero* by M. Gromov and M.A. Šhubin, **Geom. Anal. and Func. Anal.**, 1:375–404, 1991.
 - (n) *Coarse cohomology and index theory on complete Riemannian manifolds* by J. Roe, **Mem. Amer. Math. Soc.** Vol. 104, 1993.
 - (o) *Spectral theory of foliation geometric operators* by S. Hurder, unpublished preprint, 1994.
 - (p) *Pure-point spectrum for foliation geometric operators* by J. Álvarez López and S. Hurder, unpublished preprint, 1994.
 - (q) *Long time behavior of leafwise heat flow for Riemannian foliations* by J. Álvarez-López and Y. Kordyukov, **Compositio Math.**, 125:129–153, 2001.
 - (r) *The dimension of the leafwise reduced cohomology* by J. Álvarez-López and G. Hector, **Amer. J. Math.**, 123:607–646, 2001.
 - (s) *Analysis of geometric operators on open manifolds: a groupoid approach* by R. Lauter and V. Nistor, in **Quantization of singular symplectic quotients**, Progress in Mathematics, Vol. 198, Birkhäuser, Boston, MA, 2001:181–229.

4. **February 9:** *Convex hulls & spectral flow.* The notions of the algebraic hull of a quasi-periodic function, foliation index theory, and the notion of spectral flow are combined in the odd index theorem for *measured* foliations. Here are a selection of papers on these topics, including some on the applications of index theory to the “Quantum Hall Effect” which is closely related.
- (a) *C*-algebras of operators on a half-space. I* by L.A. Coburn and R.G. Douglas, **Inst. Hautes Études Sci. Publ. Math.**, 40:59–67, 1971.
 - (b) *C*-algebras of operators on a half-space. II. Index theory* by L.A. Coburn, R.G. Douglas, D.G. Schaeffer and I.M. Singer, **Inst. Hautes Études Sci. Publ. Math.**, 40:69–79, 1971.
 - (c) *Ergodic equivalence relations, cohomology, and von Neumann algebras I,II* by J. Feldman and C.C. Moore, **Trans. Amer. Math. Soc.**, 234:289–324,325–359, 1977.
 - (d) *Sur la théorie non-commutative de l’intégration* by A. Connes, in **Algèbres d’opérateurs**, Lect. Notes in Math. Vol. 725, pages 19–143, Springer-Verlag, 1979
 - (e) *Global invariants for measured foliations* by S. Hurder, **Trans. Amer. Math. Soc.**, 280(1):367–391, 1983.
 - (f) *Secondary classes, Weil measures and the geometry of foliations* by J. Heitsch and S. Hurder, **J. Differential Geom.**, 20(2):291–309, 1984.
 - (g) *K-theory of C*-algebras in solid state physics* by J. Bellissard, **Statistical mechanics and field theory: mathematical aspects (Groningen, 1985)**, Lecture Notes in Phys. Vol. 257, pages 99–156, Springer-Verlag, 1986.
 - (h) *Morse theory and von Neumann II_1 -factors* by S.P. Novikov and M.A. Šhubin, **Doklady Akad. Nauk SSSR**, 289:289–292, 1986.
 - (i) **Analysis on Foliated Spaces** by C.C. Moore and C. Schochet, Math. Sci. Res. Inst. Publ. vol. 9, Springer-Verlag, 1988. 2nd ed. pub by Cambridge University Press, 2006.
 - (j) *Eta invariants and the odd index theorem for coverings* by S. Hurder, in **Geometric and topological invariants of elliptic operators (Brunswick, ME, 1988)**, *Contemp. Math. Vol. 105*, pages 47–82. Amer. Math. Soc., 1990.
 - (k) *Cyclic cocycles, renormalization and eta-invariants* by R.G. Douglas, S. Hurder, and J. Kaminker, **Invent. Math.**, 103:101–179, 1991.
 - (l) *The longitudinal cocycle and the index of Toeplitz operators* by R.G. Douglas, S. Hurder, and J. Kaminker, **J. Funct. Anal.**, 101:120–144, 1991.
 - (m) *Gap labelling theorems for Schrödinger operators* by J. Bellissard, in **From number theory to physics (Les Houches, 1989)**, pages 538–630, Springer, 1992.
 - (n) *Quantum Hall effect on the hyperbolic plane* by A. Carey, K.C. Hannabuss, and V. Mathai, **Comm. Math. Phys.**, 190:629–673, 1998.
 - (o) *Type II spectral flow and the eta invariant* by J. Kaminker and V. Perera, **Canad. Math. Bull.**, 43:69–73, 2000.
 - (p) *Hulls of aperiodic solids and gap labeling theorems* by J. Bellissard, D. Herrmann and M. Zarrouati, in **Directions in mathematical quasicrystals**, CRM Monogr. Ser Vol. 13, pages 207–258, Amer. Math. Soc., 2000.
 - (q) *Gap-labelling for quasi-crystals (proving a conjecture by J. Bellissard)* by M.-T. Bennameur and H. Oyono-Oyono, in **Operator algebras and mathematical physics (Constanța, 2001)**, pages 11–22, Theta, Bucharest, 2003.
 - (r) *A proof of the gap labeling conjecture* by J. Kaminker and I. Putnam, **Michigan Math. J.** 51:537–546, 2003.

5. **February 22: Spectrum & asymptotic invariants.** The construction of the *coarse index* of a foliation yields a families index problem over M and, via descent, yields index classes in $K^*(M)$. When applied to self-adjoint leafwise elliptic operators paired with almost flat vector bundles, this yields a new method to estimate the spectral density of leafwise operators.
- (a) *Toeplitz operators and Poincaré duality* by P. Baum and R.G. Douglas. in **Proc, Toeplitz Memorial Conf., Tel Aviv 1981**, pages 137–166, Basel, 1982. Birkhäuser.
 - (b) *Positive scalar curvature and the Dirac operator on complete riemannian manifolds* by M. Gromov and H.B. Lawson, Jr., **Publ. Math. Inst. Hautes Etudes Sci.**, 58:83–196, 1983.
 - (c) *Eigenvalue inequalities for fermions in gauge theories* by C. Vafa and E. Witten, **Comm. Math. Phys.**, 95:257–276, 1984.
 - (d) *Conjecture de Novikov et fibrés presque plats* by A. Connes, M. Gromov, and H. Moscovici, **C. R. Acad. Sci., Paris**, 310:273–277, 1990.
 - (e) *Déformations, morphismes asymptotiques et K -théorie bivariante* by A. Connes and N. Higson, **C. R. Acad. Sci., Paris**, 311:101–106, 1990.
 - (f) *Hyper-linear cohomology and infinite groups* by A. Connes, M. Gromov, and H. Moscovici, unpublished preprint, 1991.
 - (g) *Invariance par homotopie de la signature à coefficients dans un fibré presque plat* by M. Hilsum and G. Skandalis, **Jour. Reine Angew. Math.**, 423:73–99, 1992.
 - (h) *Topology of covers and the spectral theory of geometric operators* by S. Hurder, in **Index theory and operator algebras (Boulder, CO, 1991)**, *Contemp. Math. Vol. 148*, pages 87–119. Amer. Math. Soc., 1993.
 - (i) *Coarse geometry of foliations* by S. Hurder, in **Geometric study of foliations (Tokyo, 1993)**, pages 35–96, World Sci. Publishing, 1994.
6. **February 23: Coarse invariants of chaotic foliations.** In this last lecture, we discuss the “coarse cohomology theory” of a foliation. This yields invariants of foliations based on categorical methods applied to coarse geometry. It will be calculated in a “typical” example of a chaotic foliation, the generalized Hirsch foliations.
- (a) *A counterexample to the periodic orbit conjecture* by D. Sullivan, **Publ. Math. Inst. Hautes Etudes Sci.**, 46:5–14, 1976.
 - (b) *A periodic flow with infinite Epstein hierarchy* by E. Vogt, **Manuscripta Math.**, 22:403–412, 1977.
 - (c) *Coarse cohomology for families* by J. Heitsch and S. Hurder, **Illinois J. Math.**, 45:323–360, 2001.
 - (d) *Coarse cohomology for foliations: the general case* by J. Heitsch, **Illinois J. Math.**, 46:1061–1076, 2002
 - (e) *Hirsch foliations in codimension greater than one* by A. Bís, S. Hurder, and J. Shive, in *FOLIATIONS 2005*, pages 71–108, World Scientific, 2006.

Selected textbooks on foliation theory

1. C. Camacho and A. Lins Neto, **Geometric Theory of Foliations**, Translated from the Portuguese by Sue E. Goodman, Progress in Mathematics, Birkhäuser Boston, MA, 1985.
2. A. Candel and L. Conlon, **Foliations I, II**, Amer. Math. Soc., Providence, RI, 2000, 2003.
3. C. Godbillon, **Feuilletages: Etudes géométriques I, II**, Publ. IRMA Strasbourg (1985-86); Progress in Math., Vol. 98, Birkhäuser, Boston, Mass., 1991.
4. G. Hector and U. Hirsch, **Introduction to the Theory of Foliations, Part A, B**, Vieweg und Sohn, Braunschweig/Wiesbaden, 2nd edition, 1986,1987.
5. S. Kobayashi and K. Nomizu, **Foundations of differential geometry. Vol I**, Interscience Publishers, 1963.
6. K. Mackenzie, **General theory of Lie groupoids and Lie algebroids**, London Mathematical Society Lecture Note Series Vol. 213, Cambridge University Press, 2005.
7. I. Moerdijk and J. Mrčun, **Introduction to foliations and Lie groupoids**, Cambridge Studies in Advanced Mathematics Vol. 91, Cambridge University Press, 2003.
8. C. Molino, **Riemannian foliations**, Progress in Mathematics Vol. 73, Birkhäuser 1988.
9. C.C. Moore and C. Schochet, **Analysis on Foliated Spaces** *Mathematical Sciences Research Institute Publications Vol. 9*, sec. ed., Cambridge University Press, 2006.
10. J. Roe, **Elliptic operators, topology and asymptotic methods**, Pitman Research Notes in Mathematics Series, Vol. 359, 2nd edition, Longman Scientific & Technical, 1998.
11. I. Tamura, **Topology of foliations: an introduction**, Translated from the 1976 Japanese edition and with an afterword by Kiki Hudson, With a foreword by Takashi Tsuboi, American Mathematical Society, Providence, R.I., 1992.
12. Ph. Tondeur, **Geometry of foliations**, Monographs in Mathematics vol. 90, Birkhäuser Verlag, Basel, 1997.
13. P. Walczak, **Dynamics of foliations, groups and pseudogroups**, Instytut Matematyczny Polskiej Akademii Nauk. Monografie Matematyczne (New Series) [Mathematics Institute of the Polish Academy of Sciences. Mathematical Monographs (New Series)], Vol. 64 Birkhäuser Verlag, Basel, 2004.