1. [#10, page 9] Call a subset B of a set A *cofinite* if the complement of B in A is finite. If B and C are cofinite subsets of A, prove that $B \cap C$ is cofinite.

2. [#2, page 13] Let L be a partially ordered set in which *every* subset has a top and bottom element. Prove that L is a finite chain.

3. [#3, page 13] Let $\mathbb{N}$ be the chain of positive integers, in its usual order. Is $\mathbb{N}$ complete? Is $\mathbb{N}$ complete if $\omega =$ "∞" is placed on top?

4. [#4, page 13] Let $\mathbb{N}$ be the set of positive integers and define "$m \leq n$" to mean that m divides n. Is $\mathbb{N}$ a lattice? Is it complete? If not, how could we make it complete?

5. [#9, page 13] Let L be a distributive lattice with a top element "1" and a bottom element "0". (Recall this means that $0 \leq a \leq 1$ for all $a \in L$.)
Prove: If an element $a \in L$ has a complement, then the complement a' is unique. (Recall that $b \in L$ is a *complement* for a, if b satisfies $a \cup b = 1$ and $a \cap b = 0$.)

6. [#7, page 17] Given a function $f\colon A \to A$, we write f^n for the function on A obtained by taking the composite of f with itself n times. Suppose that f^n equals the identity function for some n (one then says that f is *periodic*.) Prove that such f is one-to-one and onto.

7. [#8, page 17] As a generalization of periodic functions, we say that $f\colon A \to A$ is *locally periodic* if for every $x \in A$ there exists an integer $n(x) \geq 1$, depending on x, such that $f^{n(x)}(x) = x$. Prove that a locally periodic function is one-to-one and onto.

8. [#14, page 18] Fix a set A. For a subset $S \subset A$, the characteristic function ϕ_S of S is the function from A to the set $\{0, 1\}$ which takes the value 1 on every element of S, and the value 0 on every element of the complement $S' = A - S$. Prove, for subsets $S, T \subset A$:

(1) $\phi_{S \cap T} = \phi_S \cdot \phi_T$ (the product of the two functions)
(2) $\phi_{S'} = 1 - \phi_S$
(3) $\phi_S + \phi_T = \phi_{S \cup T} - \phi_{S \cup T}$

Optional [#11–16, page 14] These are "*" problems, and will likely require some thought to show. But the results are considered interesting problems about chains in partially ordered sets, so worth spending the extra time on. They are not required, as not even sure how hard the solutions might be; but will read and mark your solutions if you turn them in.