

1. [#1, page 26] Let A be a countable set and suppose there exists a function $f: A \rightarrow B$ which is surjective. Prove that B is also countable. (Recall that a set is *countable* if there is a bijection with the set of natural numbers \mathbb{N} .)
2. [#4, page 26] Show that the set \mathbb{N} of natural numbers can be represented as a union $\mathbb{N} = \cup A_i$ of an infinite number of disjoint *infinite* sets.
3. [#10, page 27] Let A be an infinite set, $B \subset A$ a finite subset, and $C = A - B$ the complement of B in A . Prove there exists a one-to-one correspondence between A and C .
4. [#11*, page 27] Let A be an uncountable set, $B \subset A$ a countable subset, and $C = A - B$ the complement of B in A . Prove there exists a one-to-one correspondence between A and C .
5. [#1, page 31] Prove that the set of positive real numbers has cardinal c . (Recall that c is the cardinal of the real number line \mathbb{R} .)
6. [#5, page 31] What is the cardinal number of the set of irrational numbers? Of the set of transcendental real numbers? (Recall that a real number is *transcendental* if it is not algebraic. A real number is *algebraic* if it is the solution of a non-trivial polynomial equation with integer coefficients.)
7. [#2, page 39] Let L be a lattice in which every chain has an upper bound. Prove that L has a unique maximal element; that is, a top element. (You can assume Zorn's Lemma.)