1. [#1, page 93] Prove that the space $X = \mathcal{B}(\mathbb{N}, \mathbb{R})$ of bounded sequences with the "sup norm" is complete. [This is Example 7, page 119 of Appendix 1 in the text.]

The metric is defined by, for $x = \{x_n\} \in \mathcal{B}(\mathbb{N}, \mathbb{R})$ and $y = \{y_n\} \in \mathcal{B}(\mathbb{N}, \mathbb{R})$

$$D(x, y) \;=\; \sup_{n \in \mathbb{N}} \, |x_n - y_n|$$

2. [#5, page 93] If every countable closed subset of a metric space M is complete, prove that M is complete.

3. [#6, page 93] Let $\{M, d\}$ be a metric space. If for every $u \in M$ and $\epsilon > 0$, the closed ball $D(u, \epsilon) = \{y \in M \mid d(u, y) \leq \epsilon\}$ is complete, prove that $\{M, d\}$ is complete.

4. [#11, page 93] In the space of real numbers $\mathbb{R}$, give an example of a descending sequence of non-empty closed sets with empty intersection. That is, find $F_1 \supset F_2 \supset \cdots$ where each $F_n \subset \mathbb{R}$ is closed, and $\displaystyle\bigcap_{n=1}^{\infty} F_n \;=\; \emptyset$.

5. [#12, page 93] The following functions are continuous from the real numbers to the real numbers. Which are *uniformly* continuous?

a) $f(x) = x^2$

b) $g(x) = |x|$

c) $h(x) = \dfrac{1}{1 + x^2}$

6. [#13, page 93] Let $\mathbb{R}$ be the real line with the standard metric, $d(x, y) = |x - y|$. Let $f \colon \mathbb{R} \to \mathbb{R}$ be a function which has derivative $f'(x)$ for every $x \in \mathbb{R}$. Assume there exists $K \geq 0$ such that $|f'(x)| \leq K$ for all $x \in \mathbb{R}$. Prove that f is uniformly continuous. [Hint: Mean Value Theorem]