

1. Consider a Hausdorff topological space  $T = \{A, \mathcal{T}\}$ . Consider the collection of open subsets

$$\mathcal{T}' = \{U \subset A \mid A - U \text{ is compact in } T\} \cup \{\emptyset\}$$

a) Show that  $T' = \{A, \mathcal{T}'\}$  is a topological space. (i.e., that  $\mathcal{T}'$  satisfies the axioms of a topology.)

b) Show that  $T'$  is a compact topological space.

2. Let  $X = [0, 2]$  be the closed interval in  $\mathbb{R}$ , but with a topology  $\mathcal{T}$  on  $X$  as follows:

$$U \in \mathcal{T} \iff \text{either } 1 \notin U, \text{ or } (0, 2) \subset U$$

Find the closure of the subset  $A = \{\frac{1}{2}\}$  of  $X$ .

3. Prove that there is no continuous bijective map  $f: \mathbb{S}^1 \rightarrow [0, 1]$ . [Hint: think connected.]

4. [#1, page 97] Prove that any subspace of a separable metric space is separable.

5. Let  $X$  be a locally path connected space. Show that every *open* connected subset of  $X$  is path connected.

6. [#3a, page 97] Let  $X$  be a separable metric space, and let  $h: X \rightarrow Y$  be a continuous onto map. Prove that  $Y$  is separable.

7. [#3b, page 97] Let  $X$  be a complete metric space, and let  $h: X \rightarrow Y$  be a continuous onto map. Either prove that  $Y$  is complete, or give a counter-example.

8. [#4\*, page 97] Let  $M$  be a metric space. Prove that  $M$  is separable if and only if every collection of disjoint open sets of  $M$  is countable.