

**Show all of your work!** An unjustified answer is not correct.  
Put all of your work and answers on the blank paper handed out.

**#1)** (20 points) Given the equations

$$\begin{aligned}x + 2y + 3z - 3w &= 1 \\4x + 5y + 6z - 6w &= 1 \\7x + 8y + 9z - 8w &= 1\end{aligned}$$

- Give the Reduced Row Eschelon form of the associated augmented matrix.
- Which are the free variables? Which are the dependent variables?
- Give the general solution of the system of equations.

**#2)** (15 pts) Let  $A = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 6 & 2 \\ 6 & 3 & 9 & 3 \end{bmatrix}$

- Find a basis for the **null space** of the matrix  $A$
  - Find a basis for the **column space** of the matrix  $A$
- Show your work - you must justify your answers!

**#3)** (20 pts) Use **Cramer's Rule** to solve for the value of  $z$  in the solution of the system of equations

$$\begin{aligned}x + y + z &= 0 \\x + 2y + 4z &= 8 \\x + 3y + 9z &= 2\end{aligned}$$

**#4)** (20 pts) Let  $C$  be given by  $C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 4 & 4 \end{bmatrix}$ .

- Find the inverse matrix  $C^{-1}$ . You can use either the method of Gaussian Elimination, or the cofactor method - but show all your work!
- Check your answer by multiplying  $C$  by your solution for the inverse  $C^{-1}$ .

**#5)** (15 pts) Compute the "LU" factorization of the matrix  $C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 4 & 4 \end{bmatrix}$ .

**#6)** (10 pts) Is the set  $\left\{ \begin{bmatrix} x \\ x^3 \end{bmatrix} \text{ where } x \in \mathbb{R} \right\}$  a vector subspace of  $\mathbb{R}^2$ ? Justify your answer.