

Solving a quadratic equation

a case study

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Solving a quadratic equation

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Outline

The Problem

Picturing the Solution

Some Algebra

The Formula

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2 Picturing the Solution

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A pesky problem

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Your paycheck has been held up, and they keep asking,
“Are you really a mathematician?”

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How to convince them?

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How to convince them?

What to do?

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How to convince them?

What to do?

And then the idea hits you - you'll show them you can solve a
quadratic equation!

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“Are you really a mathematician?”

How to convince them?

What to do?

And then the idea hits you - you'll show them you can solve a
quadratic equation!

If that doesn't convince the admin type, what will?

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Now, it is only a matter to select a quadratic equation which will impress them.

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Now, it is only a matter to select a quadratic equation which will impress them.

$$1 \quad x^2 = 0$$

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Now, it is only a matter to select a quadratic equation which will impress them.

1 $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)

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Now, it is only a matter to select a quadratic equation which will impress them.

1 $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)

2 $x^2 - 2x + 1 = 0$

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Now, it is only a matter to select a quadratic equation which will impress them.

- 1 $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)
- 2 $x^2 - 2x + 1 = 0$ (more of the same)

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Now, it is only a matter to select a quadratic equation which will impress them.

- 1 $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)
- 2 $x^2 - 2x + 1 = 0$ (more of the same)
- 3 $x^2 - 3x - 1 = 0$

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The Formula

Now, it is only a matter to select a quadratic equation which will impress them.

- 1 $x^2 = 0$ (nah, too obvious. it would be shameful if this worked)
- 2 $x^2 - 2x + 1 = 0$ (more of the same)
- 3 $x^2 - 3x - 1 = 0$ (sort of fancy... just right!)

Grab your calculators:

A picture may be worth a thousand words, but is it worth a thousand bucks?

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Grab your calculators:

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A picture may be worth a thousand words, but is it worth a thousand bucks?

Let's try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$

Grab your calculators:

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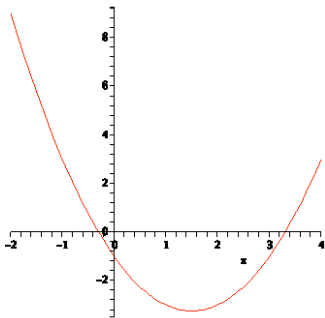
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A picture may be worth a thousand words, but is it worth a thousand bucks?

Let's try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$



Not even close...

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“You want money for your one lousy graph?”

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“You want money for your one lousy graph?”

“Give the solution to 10 decimals, and we’ll show you the money!”

Not even close...

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“You want money for your one lousy graph?”

“Give the solution to 10 decimals, and we’ll show you the money!”

“Oh, for @#%& sake!”

factor, factor, complete...

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$$0 = x^2 - 3x - 1$$

factor, factor, complete...

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$$0 = x^2 - 3x - 1$$

$$0 = x^2 - 3x + (-3/2)^2 - (3/2)^2 - 1$$

factor, factor, complete...

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$$0 = x^2 - 3x - 1$$

$$0 = x^2 - 3x + (-3/2)^2 - (3/2)^2 - 1$$

$$0 = (x - 3/2)^2 - 9/4 - 4/4$$

factor, factor, complete...

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The Formula

$$0 = x^2 - 3x - 1$$

$$0 = x^2 - 3x + (-3/2)^2 - (3/2)^2 - 1$$

$$0 = (x - 3/2)^2 - 9/4 - 4/4$$

$$0 = (x - 3/2)^2 - 13/4$$

Progress

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Now let's solve it:

$$0 = (x - 3/2)^2 - 9/4 - 4/4 \implies (x - 3/2)^2 = 13/4$$

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Now let's solve it:

$$\begin{aligned}0 &= (x - 3/2)^2 - 9/4 - 4/4 &\implies (x - 3/2)^2 &= 13/4 \\ & &\implies (x - 3/2) &= \pm\sqrt{13/4}\end{aligned}$$

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Now let's solve it:

$$\begin{aligned}0 &= (x - 3/2)^2 - 9/4 - 4/4 &\implies (x - 3/2)^2 &= 13/4 \\ & &\implies (x - 3/2) &= \pm\sqrt{13/4} \\ & &\implies x &= 3/2 \pm \sqrt{13/4}\end{aligned}$$

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Think this is enough to get the money?

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Now let's solve it:

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Think this is enough to get the money?

Not likely...

Pay Up!

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There are two solutions:

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There are two solutions:

$$x = 3/2 + \sqrt{13/4}, \text{ or}$$

$$x = 3.302775637731994646559610633735247973125648286922623$$

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There are two solutions:

$$x = 3/2 + \sqrt{13/4}, \text{ or}$$

$$x = 3.302775637731994646559610633735247973125648286922623$$

and $x = 3/2 - \sqrt{13/4}, \text{ or}$

$$x = -0.3027756377319946465596106337352479731256482869226$$

Mathematical Proof

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The final proof that we are Mathematicians?

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The Formula

The final proof that we are Mathematicians?

Give them the Magic Formula,

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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The final proof that we are Mathematicians?

Give them the Magic Formula,

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and tell them to try this first next time...